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THE PROBLEM OF PHYSICAL EXPLANATION

STATEMENTS of the meaning and purpose of physical explanation are numerous. Every philosopher who has written on "Inductive Logic" has in some manner described or circumscribed the problem of physical explanation. The present paper is not a review of existing knowledge or opinions. It was written with an acute consciousness of the incapacity of most theories of explanation to provide grounds upon which modern hypotheses may stand, and it attempts to formulate a concept and an understanding of physical explanation which is, as far as logical principles permit, in conformity with *modern* views.

Scientists often satisfy themselves, and dispense with the complexity of the problem, by stating that explanation is nothing but a search for causes. But the semblance of truth which attends this assertion proves, on closer analysis, to depend on a delusion about the meaning of "cause." Cause, in the parlance of those who are accustomed to precise manners of speech, is an observable fact which, in a series of sense perceptions, invariably occurs as the antecedent to a definite set of other observable facts, called consequents. Hence, when in the course of this inquiry I speak of the cause of any phenomenon, I do not mean a cause which is not itself a phenomenon. A necessity of transcending into a metaphysical world of non-perceptible entities does not exist for the purpose of

scientific investigations; nor will any doctrine concerning the transcendental nature of phenomena be found in the following remarks. Causes are definite and discernible components in the course of natural events, data of nature which need not be inferred by speculation. However, explanation deals with entities which are not, in general, observed, and can therefore not properly be called causes.

The term "nature" will occur more frequently than any other in the subsequent discussion; hence it should be most clearly defined, and stripped of all colloquial implications which do not belong to its strictest meaning. Nature, in the most noncommittal sense, is the totality of sense impressions, this latter term including observations of a systematic and scientific character. It comprises nothing external to the human mind and postulates no things in themselves. Nor does it involve any assertions with regard to the non-existence of things in themselves. Data of nature will be understood to be parts of our consciousness, and should imply no suggestion as to their origin. The very word datum is misleading. For if our perceptions be gifts we might expect to find a giver, which some recognize in an objective nature, others in a divine agency. But inductive experience teaches nothing about a giver, nothing about the origin of sense impressions; it leaves us in the bare state of *having* cognitions, without even an indication of their whereabouts. The term "habita" instead of "data," the use of which was advocated by a philosopher whose name I have forgotten, would appear preferable in view of this condition. But its introduction would constitute too radical a break from current terminology to warrant its usefulness. The word datum will be employed in this specified sense.

We have seen that nature is the sum total of data. However it is a common practice of the mind to project

these data into an external space and time, to endow them with substantiality and construct an objective nature which is a thing in itself. This process is so universal that a few critical remarks need be made concerning its legitimacy. Let us analyze it briefly and consider some of the steps. Certain experiences cause us to form inseparable associations between sense perceptions, persuade us to become convinced of their existence independently of our perceptions, which we finally accustom ourselves to look upon as accidental. We have invested the possibility of sensation with the quality of permanence. So far an objective nature is a phychological structure. Belief in an external world is simply the belief in a permanent possibility of sensation. It is important to recognize that we need not go farther than this to establish a ground for science. But the realist will refuse to stop at this point. His next step will be to class these permanent possibilities as existences distinct from our sensations, and this may be proper if "existence" is not understood in the medieval ontological sense; but if he thinks of them as generally distinct, as causes of our sensations, he is deluding himself by a faulty use of what is called causation. The law of causality may not derive its unconditional binding force *from,* but certainly manifests it *in* actual experience, and every conceivable case to which it is applicable consists of two sets of data of which one may be inferred if the other is known. Now an objective nature is not a determinable factor of experience, and we have no evidence that it comes within the range of entities over which causality has legislation. It is for this reason that the argument fails—that causality can never form a bridge between the known and the ultimately unknowable, and that an objective nature, if it exists, is forever unintelligible. Another feature which strengthens the

fallacy of an objective nature is the tendency of our mind to ascribe greater immediateness, not to use the word reality, to notions which present themselves more readily than others. For instance we are tempted to consider matter as more immediate and real than, for example, energy or an electric field. But it is evident that this propensity is caused by the constitution of our senses, or mind in general, and must therefore be regarded as accidental. Indeed it will appear that the algebraic relation $pq - qp = \frac{h}{2\pi i}\varepsilon$ may be just as essential a component of reality as is the notion matter. But this consideration must be deferred until later. The recognition that the realistic appeal of certain natural data is largely dependent upon our modes of comprehension deprives naive realism of a considerable portion of its persuading force.

If nature has no intelligible objective counterpart, and if its data have no generic agency, how can the uniformity of nature be accounted for? I am using the term uniformity in John Stuart Mill's original sense as referring to the prevailing regularities encountered in the order of data. The occurrence of laws in nature is indeed one of the strongest arguments against the ideal immanence of physical science. Although the validity of numerous laws proves nothing by itself about the objective reality of nature, we do find it less plausible to assume a great number of independent uniformities in our experience than to assign to an objective nature equally many *properties* from which the uniformities in experience follow generically. The simplest and most elegant way out of thist difficulty is that taken by the extreme idealist who interprets all laws as categories projected into an imaginary external world from within. But we do not wish to

make any postulate with regard to an external world as a thing in itself, not even its non-existence; we merely wish to record our absolute ignorance concerning it. The only thing we have shown is that there is no causal connection between it and natural phenomena, and that, if there be any link, it is unintelligible. Moreover, we wish to avoid the dilemma wherein the idealist finds himself when he attempts to understand the separate existence of conscious beings. It remains to be seen whether or not the separate uniformities impressed upon us by the very data of nature can be resolved into few simple assumptions. If it is possible to reduce them to one or two axiomatic principles our surprise at finding nature so peculiarly obedient to a multitude of laws will be much diminished, and the tendency of assigning independent properties to an objective thing in itself behind nature, in order to produce these laws, will have lost its power. This reduction, I hope to show, can be performed.

In any group of sense perceptions there will, in general, be definite features of comparison. The group, first chaotic, displays similar traits, allowing the intellect to begin its ordering action. Such resembling features may be termed synthetic properties of the complex of sense awareness. They themselves constitute no laws; it is the regularity of their occurrence that forms a law. This regularity, or uniformity of behavior, imposes a very definite restriction upon nature, which is not inherent in its concept; it makes nature subject to a form of predetermination which permits us to describe its unknown data with some expectancy of correctness. The character of this restriction might be of great complexity; the predetermination might exist in a multitude of ways. Instead, we find that the regularity is of the simplest imaginable type, namely, that a given set of natural events

determines a definite set of others whenever the elements composing the primary set combine to restore it. This fundamental axiom, it will be observed, is more general than the principle of causation, for it establishes the possibility of numerical relations and geometrical intuitions which could be deduced only improperly from the law of causation. It will be referred to in the course of this discussion as the principle of consistency of nature, and shown to include the principle of causation. I find it difficult to formulate this latter in a manner that is universally acceptable, in view of the circulation of such vague and divergent notions as are expressed by: *"Causa aequat effectum;"* "Everything has a sufficient cause;" "Everything has a beginning in time." The general recognition seems to be the determinateness, the impossibility of arbitrary intrusion into the course of events. This, however, is precisely the content of the principle of consistency of nature if the relation between the primary and the determined set of events be one of succession. It may be remarked parenthetically that this same principle yields numerical and geometrical laws if both sets are coexistent. With more particular reference to physics the principle of causation may thus be stated: No matter at what time an experiment be performed, its result is the same if the original conditions are identical.

Perfect identity of causal conditions can never be achieved. Hence, if the principle of consistency be regarded as an *a posteriori* datum it can be established only as an ideal abstraction from an approximating, ever recurring experience. As such, it would exhibit all the defects of an experimental discovery, would carry with it no assurance of universality or the necessity of its continual recurrence.

On the other hand, it may be supposed that it is a

category of *a priori* rank, a fact which lies at the very basis of experience as the condition for its possibility. The decision has little to do with the application of the principle; the physicist, in attempting to understand the nature of his explanations, need not commit himself on this point.

A difficulty which the last mentioned form of the principle of causation makes no attempt of concealing lies in the rôle which "time" plays in it. It would seem at first that the abolition of absolute time and the dissolution of the concept of simultaneity by the theory of relativity confronts us with a grave problem concerning the truth of the principle. This matter does demand attention. However, since it is but loosely connected with the topic of this paper, I shall not deal with it in detail, and merely state that the principle is in conformity with relativity considerations as far as I have been able to extend the analysis.

Bridgman, in his *Logic of Modern Physics,* refers to a fundamental postulate which he calls the "principle of essential connectivity" and phrases in some such way as this: If two apparently similar isolated systems start from the initial conditions, and differences develop in their behavior, these differences are evidences of other previous differences. It is clear that this follows as an immediate consequence from the principle of consistency of nature; and it appears to me to be less general than the latter. Moreover, he appears to make the validity of the principle of causation dependent upon the existence of linear differential (or difference) equations to describe the phenomena. It is certainly instructive to point out that in case of non-linear equations we may be unable to carry out the causal analysis because of the unmanageable mathematical complexity of the problems. That may even be true in the

case of linear equations. But the principle still applies if our analysis is impossible; the effects of the partial events need not be additive, the equations need not be linear. In general, a complex effect may not be representable as the superposition of elementary effects—yet it is still unique, and the principle of causation requires no more.

The preceding remarks are likely to appear out of place, since they have but little to do with the topic of this discourse. Yet I must contend that an analytic inquiry into the meaning of "nature" and "natural law" is essential in any attempt to rise to as general as possible a view of the problem of physical explanation. It is indeed my effort to outline its widest aspects, for any restrictive definition would exclude much that is valuable in the field of modern hypotheses.

The first step in our constructive program is to acquit ourselves of the accusation of doing unnecessary labor when we frame explanations. Let us, then, pause for a moment and contemplate the reasons for the necessity of explanation. If physics were merely a science of measurements there would be no room for explanatory investigations; the theoretical physicist would lavish his energies in diluting concrete facts. But it is the characteristic of a true science that it proceed in close association with philosophy, that it be conscious, in every step of its development, of its relation to the fundamental principles of logical reasoning. Why do we not content ourselves with a mere description of nature, renouncing all attempts to explain it in terms which it does not itself offer without the intervention of reason? The answer is that the context of sense awareness is unrelated, and that we are compelled by the very constitution of our organs of recognition to conceive of events as continuous, as related. The unsophisticated mind experiences considerable dis-

satisfaction when it observes that gradually varying causes produce effects completely out of harmony with their gradual changes. Charge two spheres by constantly increasing their potential. During the first part of the process nothing directly observable occurs; then, suddenly, a violent discharge appears. Heat a substance slowly, and the effects will be consistent only up to a certain limit; then an unexpected change in phase is witnessed. Physics is full of instances of unrelated variations. If it were not, the desire for explanation in notions other than purely descriptive would probably never be felt. Again, there is the transition from one phenomenon to another which appears to abrupt to gratify our longing for coherence. Let an ordinary piece of steel act upon a bit of iron and there will be no effect. Another piece of steel, similar in its external properties to the first, will attract the iron. The unsophisticated observer is puzzled. The principle of consistency of nature, which he unconsciously embraces, compels him at once to formulate theories with regard to the internal properties of the second piece of steel: he is indulging in physical explanation. To investigate further this basic urge of human nature, which causes us to work for unity and coherence, to yield uncritically and without reserve to its universal appeal, would lead us into interesting speculations. But for the purposes of this discussion it is sufficient to state its existence, and to acknowledge its constraining power.

We are now about to realize the function of physical explanation. It is clear that its purpose is more fundamental than to reduce the number of different concepts needed in the description of nature, or to serve as a guide in experimental investigation. For it falls short of accomplishing the former task; it introduces new concepts in addition to the ones required to comprehend our experience,

which still remain necessary. The latter purpose, that of guiding the experimenter, it incidentally achieves, but its mission is far nobler than its capacity as a handmaid of observation. In the remainder of this discussion I shall try to analyze the process of explanation into its elements, starting with simple forms of explanation, determining their special features. Then it will be my aim to widen the meaning of the term, and finally to lay down some requirements which it must always answer. In the end it will be desirable to survey the field of physics and to determine in what respects existing theories conform, or do not conform, to these requirements. In following this course we shall observe that the meaning of physical explanation has undergone distinct modifications, the most radical of which is taking place in modern physics.

Whenever we make any attempt to explain an occurrence, that is a set of observable events or phenomena, we associate with each phenomenon in nature some definite concept, or relation between concepts. This statement sounds very abstract and almost inane. Yet it is necessary to make it in such a general form in order to avoid the unpleasant obligation of qualifying it later. To be sure, it means that we establish a correlation between nature and a certain system of thought, called the explanation of nature. The elements of this system of thought, to which I have referred as definite concepts, or relations between concepts, I shall take the liberty of calling explanatory symbols, or simply symbols. Up to this point we have discovered nothing, either in nature or in the constitution of our mind, that would restrict our choice of possible symbols. Nevertheless we find that in most explanations the symbol is chosen to be some imaginative process, mostly capable of visualization. For the sake of simplicity we shall permit ourselves for the present to think

in terms of such processes, although it will appear later that there is not only no necessity for such restriction, but that, under certain conditions, it may actually be desirable to abandon it. As examples of such correlation we have the measurement of heat on the one hand, rapid motions of idealized spheres—called molecules—on the other; or the appearance of a spectral line on the side of nature, and the electron transition as its physical explanation. Our problem will be solved when the nature of the correlation in question is determined. The common misconception that it be a causal relation has been guarded against at the beginning. It is here to be noted that an ill defined notion of causality has done much to confuse the minds of scientists, and has unhappily suppressed the appreciation of the function of physical theories and hypotheses. Motion of ideal entities is *not* the cause of heat, if the term "cause" is to have any meaning at all. Not even the invention of such a vague and misleading phrase as "hidden cause" or "underlying cause" enables us to make this statement in the affirmative. The cause of the heat, or, better, the body's being hot, is the process of heating it, a phenomenon with which nature presents us directly. The assignment of the symbol (motion of molecules) to the observable phenomenon of heat is an arbitrary act of reasoning, the justification of which we have to discuss and establish.

Another very common view is that the correlation is one of identity. The imaginary process is said to be the same as the occurrence explained. In fact it seems as though this were the universal credo of physicists today. Almost every experimenter believes sincerely that the emission of a spectral line *is* the same as the transition of an electron. An analysis of this situation would involve the problem of objective reality, which I am carefully avoiding

because of its complexity, and, fortunately, we need not consider it to perform our task. One thing is certain: namely, that the indicated attitude is extremely useful as a working hypothesis and probably much to be encouraged. Yet the logician would assert that the imaginary process and the natural event are not identical; at least we should have no means of proving the identity. And if we still insist on calling heat and kinetic energy of molecules identical, we should be open to the accusation of using the word identical in a sense differing from its original and rigorous meaning. Granting, then, that it is advantageous to think in many instances, of our explanation as the event to be explained, we must maintain from our analytical point of view that identity fails to designate the correlation we are seeking. We are forced simply to call it, without pretense of further specification, a definite correspondence.

When the illusion has been destroyed that explanation stands to natural event in a causal or identical relation, the explanatory symbol loses much of its substantiality and its immediate appeal, but not its value. Disregard of objectivity causes us to feel more free in selecting symbols: we need no longer have scruples in exploring the regions of abstract thought in our search for suitable symbols. It is true that visual processes are desirable for their convenience, but it may happen that we are driven to rid ourselves of the shackles which they impose. Then it will be imperative to abandon convenience and to accustom ourselves to more arduous methods of reasoning.

The wider the domain from which we may chose symbols for the explanatory representation of experience, the more necessary becomes a criterion for their selection. There is evidently an infinite number of imaginative processes that can be associated with any event. Why do

we prefer a single one to all others? And, after having selected one, whence do we draw the evidence for the correctness of our choice? One fact is recognized at once: Whenever a new field of science, presenting but a single, or few observations has been discovered, there are a multitude of conflicting systems of explanation. An increase in the number of known phenomena is always accompanied by an elimination of hypotheses. It is safe to say that if in physics we had knowledge of but one single fact, its explanation would be meaningless, impossible, and no one would feel the need of it. Physical explanation has meaning only if there is a group of observations to be explained. The symbols used to construct a theory covering this group of observations must in the first place be limited in number, and secondly, they must be related. The significance of these two restrictions will be more clearly understood if we analyze the example already referred to. Kinetic energy of molecules is a combination of symbols, or, in this case, the term that calls to our minds a set of imaginative processes designed to explain heat. We consider it a satisfactory explanation. Suppose we attempt to find a few reasons of a general character why the kinetic theory is preferable to the old caloric theory, for instance. We find immediately that there are in our experience with heated bodies certain changes, such as freezing, fusion, and evaporation which demand the introduction of symbols other than are involved in the existence of a caloric, and entirely unrelated to it. The kinetic theory, however, has a sufficient number of related symbols to cover these phenomena. It may be said that the kinetic theory is more consistent, which is but another way of stating that its symbols are related. Moreover, if any theory needed the assumption of moving molecules, of the existence of a caloric, of the attraction

between molecules, and perhaps a dozen other assumptions, we should reject it without test for it is wasteful of symbols.

These remarks are still much too general to be of assistance in determining which individual symbol within a group is to be associated with a given phenomenon, and before this has been specified there is little meaning in other considerations. Again let us look to the kinetic theory for elucidation. To be more specific, let us consider a gas. Under certain controllable conditions it rises in temperature, that is, a change occurs that can be measured. To this phenomenon we assign a symbol, tentatively that of "added swiftness of molecules." If the gas be enclosed in a vessel, another phenomenon may be observed and measured at the same time: that of increase in pressure. There is distinctly no similarity in the outward appearance of these two effects; they seem perfectly incoherent. Yet they have the same cause. The explanatory symbols, by their very reason for existence, must preserve the relatedness of the two events which nature obscures. Now we know from elementary observations that a moving particle has momentum, and that a change in momentum produces a force. Such change in momentum takes place at the boundary of the vessel. Thus we are led to associate with the observation of change in pressure the symbol change in momentum of molecules. On the part of nature we have two occurrences, quite different in character, but joined by the similarity of their causes. To them we assign two symbols so related that one is implied in the other. Take now two different gases and allow one to pass into the other. We are still dealing with gases, hence at least part of the causes are the same as those previously considered. We observe the phenomenon of diffusion, an effect which no longer

resembles any of the other two. Yet, I repeat, one of the causes is the same, for we are using the same gas as before. But diffusion is explained by simply assuming the properties of molecules belonging to different gases to be different. We are introducing a new symbol, or what amounts logically to the same thing, modifying slightly an old one and we employ it to account for the effect of diffusion. Thus we might continue. We discover more and more phenomena, completely different in appearance, but related by the similarity of their causes. Our system of explanation preserves the continuity or relatedness of the causes; it comprises only symbols of such character that a transition from one to a related one corresponds to the step from one occurrence in nature to another one which has a similar cause.

This situation reminds one indeed of the process which mathematicians call mapping. In choosing this process to illustrate the meaning of physical explanation I am fully conscious of the inadequacy of the use of pictures to represent abstract facts. But this illustration is so formal and devoid of accidental details that is not likely to distort our analysis. We shall henceforth frequently refer to it. Think of the data of nature as points in nature space. They will be dense in certain regions, sparse in others, depending on the care and thoroughness with which the region has been investigated experimentally. Each point is a known fact, its coordinates may be descriptive properties. Now experiment reveals but single phenomena, never any relations between them. Hence, however densely the points may be clustering in a certain domain, their distribution will always be discontinuous. As Bridgman expresses it: every experimental discovery has a penumbra of uncertainty about it. The extension of this penumbra may be reduced by increased pre-

cision of measurement but it can never vanish completely. This is quite true in nature space. Yet while Bridgman satisfies himself with the existence of this uncertainty as a deplorable but inevitable fact, *we demand that physical explanation abolish it.* We desire a representation of empirical knowledge which is, in terms of our graphical illustration, continuous. Therefore, we require a correlate to nature space, which we may call explanation space. In this explanation space, each point is a symbol in the previously defined sense, and each symbol corresponds to a point in nature space by the condition that a transition from one point in nature space to a nearby point be represented by a small and continuous variation in the associated symbols. A certain set of symbols, such as that involved in the kinetic theory, may, or may not, suffice to explain the entire domain of physical data. This matter will be discussed a little later. For the present we shall only point out this two-fold possibility, and note that such a determined set of symbols defines, in our explanation space a certain mode of representation, which may possibly have to be changed as we proceed to explain another field of physical data.

At this point it becomes necessary to remark on the origin of the symbols. I fear that I have given the impression that they are, or might be, entities entirely distinct from natural phenomena. The extreme idealist would have it so. Surely, it would be beautiful and conservative of much mental effort if we could split the totality of human knowledge into two distinct categories, one comprising nothing but natural data and the other one our speculations about nature unstained by the admixture of sense impressions. Unfortunately, however, we can not even gain natural data without the aid of intellectual processes, and we can not think about nature in terms completely free

from reference to elementary external events. The greatest problems in the theory of knowledge arise from the diffusion of these two classes of things. We are compelled, then, to choose our symbols of explanation in accord with this situation. A group of purely logical postulates is never to be selected as a set of symbols, for there would be no intelligible connection between the explanatory theory which they constitute, and nature's data. Hence, in every mode of representation of our discoveries about nature, there must be found some symbols which are directly derived from nature by observation. It is to be observed that even the most abstract theories of modern physics conform to this requirement. This statement is not really a restriction upon the choice of symbols; it is merely the condition that they be intelligible, that they have meaning.

Perhaps the grandest and most successful attempt at physical explanation is the work which has led to the mechanistic view of the universe. It will be interesting to devote our attention to it for a little while. First of all we must understand that the mechanistic view is by no means the same thing as the tendency to design mechanical models. The latter has not been confined to the mechanistic theory; it is to be found even in theories most remote from all attempts of mechanical explanation, and merely reflects a prevalent habit of thought. If we wish to find the mechanistic view in its purest form we must look to Helmholtz, I believe, who expressed it in a classical manner in his famous lecture "On the Conservation of Force," held before the Physical Society of Berlin in 1847. He designates as the aim of physical science the explanation of all natural phenomena in terms of motions of discrete material particles subject to the action of attracting or repelling central point forces, whose magni-

tudes are functions of the distance between the particles. *There* is a most clearly defined and coherent set of symbols, which exhibit the properties of which we have already spoken. Their number is small; they are related; and at least some of them, such as motion, material are immediate natural data, whereas others, such as particle, point forces, are idealizations which are not found directly in nature.

Suppose that we investigate the range of validity of this system of explanation. Mapping the points in nature space we begin with mechanics and find that they are all represented by one continuous curve in the associated explanation space. We pass from one phenomenon to the next by changing slightly the combination of our symbols. The same curve carries us without a break into the regions of sound and heat and a good distance into electricity. Points in nature space can be connected, that is, experimental data can be combined, in an unlimited number of ways, each particular way corresponding to a curve in explanation space. The number of possible curves in a certain portion of the latter space will depend upon the density of points in nature space; but we postulated that explanation space be continuous, hence there must be points between the curves. And it frequently happens that, noticing an interesting point somewhere between two curves to which a point in nature is not yet known to correspond, we look for it in nature space and find it there.

When we realize that our curves stop somewhere within the region of electricity, we are about to make another interesting discovery. There are phenomena in nature that can not be mapped as long as we use the same simple mode of representation. Incidentally they have to do with propagation of electric disturbances. Here, our symbols

fail to be applicable, for they would mislead us into expecting that the energy transmitted should be associated with material particles. (I am professing at present no knowledge of the quantum theory). But to this conclusion, which is a point in explanation space, there corresponds no point in nature space. Now it is within rights to combine the symbols in such a manner as to invent an ether, although some might doubt that this can properly be called a *small modification* of the elementary symbols. Yet even then we do not find the complete correspondence which we are seeking. To be sure, we can construct it after the fashion of McCullagh and Lorentz, endowing the ether with rotational elasticity. Indeed, it is interesting to note that Henri Poincaré has shown that every natural phenomenon may be explained by the mechanistic theory, even in an infinite number of ways. Why is it, then, that we feel strongly disinclined to accept as a physical explanation an ether with connected molecules containing three mutually perpendicular gyrostats in uniform angular motion with little angels flying around and keeping them properly lubricated? At first one would probably answer that the set of symbols involved is too artificial and complicated. Yet we must reject that answer if we wish to save some of the most widely accepted theories of to-day. It is highly unscientific to reject a thought because it is inconvenient. The correct answer is that there is nothing in nature to correspond to the added explanatory symbols, except the single phenomenon of propagation of electric waves. We are introducing into our explanation symbols capable of variations for which nature has no independently variable attributes. This condition may be roughly designated by saying that the system of symbols has more degrees of freedom than its natural correlate. Here we recognize another essen-

tial requirement to be imposed upon any satisfactory system of explanation, a criterion for the validity of a physical theory. In the instance under discussion, a defender of the mechanistic view might help himself out of his difficulty by fixing rigidly the axes of the molecular gyrostats, and by postulating that they be forever free from mechanical defects and inaccessible to experimental change, thus reducing the degrees of freedom permitted by the explanatory symbols. But if such practice were considered legitimate, it would violate the first requirement, namely that our symbols be few in number; for its uncontrolled use might increase their number without bounds. Consequently we are forced to admit the failure of the mechanistic theory of explanation in the domain of electricity.

Now we are in possession of several criteria for the fitness of a theory to explain occurrences in nature. They were derived by analyzing the meaning and purpose of physical explanation. Let us muster them in review, before we go on. At first we found that the explanatory symbols must be limited in number, and kept so by all permissible procedures of extending and elaborating on the system of explanation. At the same time we recognized that the symbols had to be related. Then there is the requirement that the set of symbols have no more variable attributes than the phenomenon it represents in nature. Last, but most important, perhaps, is the peculiar correspondence between events and the symbols used for their explanation, a correspondence which was made clear by reference to the properties of the curves in explanation space, which had to be continuous, and were made to cover completely a certain domain of space.

This latter correspondence will occasion some further scrutiny. The breakdown of the mechanistic theory

shows that a system of explanatory symbols, which has been very successful in certain fields of physics, may fail to be applicable to other groups of phenomena. In other words, our points in explanation space cover completely and continuously a definite portion of it. Outside of this part of space we can not go, unless we are willing to use a different mode of representation, that is, a different set of symbols. This situation is unsatisfactory indeed, and it caused considerable alarm at the time of its discovery. Even now, most of us are still inclined to view it with some sort of apprehension. It is my personal belief that this attitude is not merely the result of our surprise at learning a few novel and unexpected aspects of nature, but that it contains a justifiable feeling of doubt as to whether the possibilities of finding an explanatory system covering all phenomena have been exhausted. Considering how wide a field of choice we have for our symbols, and how few efforts have been made to extend a theory over all ranges of natural events, we should certainly be convinced that we need not yet commit ourselves to the necessity of explaining nature in terms of different, non-consistent theories. There is much opportunity for profitable and constructive work in theoretical physics, and some interest in achieving unity and coherence in the various systems of explanation has fortunately arisen.

However, it is not all certain that such unity is possible. Does, in that case, physical explanation lose its meaning? If it is to retain its meaning, the correlation between the points in nature space and the points in explanation space must be definite, that is to say, the event corresponding to a certain possible variation in the symbols must be definable and unique. This is equivalent to a point to point correspondence between the data of nature

and the points in explanation space. Two ways are imaginable in which the correspondence might fail to be definite. First, one explanation could cover two or more phenomena. This case may be dismissed without further consideration, for it will always appear that, whenever this difficulty exists, it is due to a fallacy, caused by an insufficient analysis of the symbols involved. The other possibility is more serious: one phenomenon might be represented by several distinct sets of symbols, or, in terms of our favorite simile, one point in nature space might correspond to more than one in explanation space. This situation can not be tolerated, for in every conceivable form of this dilemma it is found that, in explanation space, points in the immediate neighborhood of those under discussion can be selected which designate conflicting phenomena in nature. To refer to one instance: both the kinetic and the caloric theory explain certain observable facts about heat. But the first would demand that heat be always associated with matter, whereas the latter would carry with it the possibility of its isolation, two things which can not both be true.

If the reasoning that has led to this particular conception of physical explanation is at all conclusive, we may formulate some ultimate inferences which are interesting because of their immediate applicability to modern theories. There is a universal desire that a system of symbols be found which will serve to represent all physical phenomena. This may be said to be the aim of scientists. But it may be beyond our power to design a system of this character; the final verdict in this matter is nature's. Even then physical explanation has meaning as long as there is a point to point correspondence between explanatory symbols and natural events. If this correspondence breaks down, physical explanation is impossible.

Fortunately, we have as yet no cause to believe in the ultimate failure of this correspondence, although there are instances in physics at the present in which it does not exist.

Such conclusions as these might be regarded and applied as clearing and ordering agencies among modern physical theories. It would be possible and interesting, perhaps, to base upon them a critique of scientific hypotheses. However, I have not the space to survey the whole field of physics, nor would I be capable of cataloguing exhaustively all the theories that would enter into the discussion. Permit me to criticize, from the point of view of this analysis, only one outstanding theory, the quantum theory of popagation of light. Can we consider it as satisfactory, or not? To be sure, the quantum theory, in its more general form, does not satisfy our desire for unity, but we saw that this was not absolutely necessary. There is a sudden break in the mode of explanatory representation when we pass from the emission of light to its propagation. Two distinctly different sets of symbols are required for these naturally related events. Yet this is permissable since we postulate continuity of explanation space only in discrete domains. Hence if the application of the quantum theory were confined to explain problems of emission and the wave theory to account for those of propagation, both would be perfectly legitimate theories, in spite of the fact that we know of no connecting link between them. However, as soon as the quantum theory is called upon to explain the phenomenon of the propagation of light, and it functions, it ceases to preserve its character as a satisfactory system of explanation.

From our present point of view, what is the difference between a theory and a hypothesis? The answer is easily

given: a hypothesis is a set of explanatory symbols valid over only a small range of natural observations. It may be obliterated by another hypothesis which expands and covers its region, which we should then be inclined to call a theory. As the range of application determines the value and power of a set of symbols, we are quite justified in attaching greater importance to a theory than we give to a hypothesis. Yet there is no essential difference in the nature of these two constructs.

Modern physics abounds in theories of limited ranges of application. In reflecting upon this condition, and observing the insistence with which each investigator elaborates upon and extends his theory of explanation, we might feel that science is no longer inspired by the hope of finding a single, consistent theory to cover all of nature's data. But there is a very promising feature about most modern theories: most of their explanatory symbols are purely formal. This does not make them better *per se,* of course, yet it entails a tremendous advantage, namely the possibility of merging one theory with another. Had the ψ-function of Schroedinger's theory represented something very definite, its meaning would have precluded all chances of fusion with the theory of Heisenberg. Page's success in uniting the theory of special relativity with that of electrodynamics might not have been so complete if the relations of relativity—its explanatory symbols—were less abstract and general. It was indicated before that there is no reason why the ultimate system of explanation should not be of an abstract and formal character, as long as some of its symbols relate to actual experience. Abstract explanations are to be encouraged because of the greater ease with which they combine.

The reason why many physicists are willing to accept an abstract theory is quite apparent, and has been alluded

to before. They find it less convenient, more elusive, than their accustomed combination of workshop models. Some would emphatically protest, pray to their little idol called truth or reality, and recite their well learned creed denouncing abstract things as unreal. But I have not yet heard of any definition of explanation in terms of what is real; moreover, if we knew reality, we should not bother about explaining it. Reality is not primary to explanation; it can only be said to be identical with the most satisfactory system of physical explanation. We shall discuss this matter again shortly.

Anschaulichkeit, picturesqueness, as Biggs wants it translated, is not a necessary attribute of explanatory symbols. The question concerning the rôle of time and space in modern theories has become quite acute, in particular their continuous, or discontinuous nature. It is clear at the outset that every system of explanatory symbols will involve time and space in some manner, for if it did not, it would not be intelligible, and a relation to actual experience could not be found. One sometimes hears the assertion that the modern quantum theory has proved time, or space, or both, to be discontinuous. This I wish to show to be a fallacy, at least an incorrect statement of the case. I know that I run the risk of being ousted from the association of fashionable physicists when I confess the heretical belief that time and space are not wholly facts of experience. I shall not go so far as to say with Kant that they are forms of intuition of pure reason, completely independent of being influenced by empirical recognitions. Nor can I follow Einstein and others in their rather uncritical acceptance of the dogma that they represent relations we have learned from nature. I hold that they contain an element

which is uncontrolled by natural data, without which experience would be impossible.

It is certain that there must be traits in our knowledge which mark the peculiar organization of our mind, notions that would be different if our modes of comprehension were different from what they chance to be. As slight a change in our mental make up as a shift of the sensibility of the eye from light to X or γ-rays would occasion a complete change in our conception of nature; what would happen if our ways of reasoning were altered can not be foreseen. You will probably object that our mind is but a datum of nature. Yes indeed, but you will forever be unable to understand the possibility of experience unless you assume the existence of a mind prior to experience. There are other reasons, but this is the most pertinent one for assigning to the mind a position unique with regard to occurrences in nature. Time and space are among the elements whose form is conditioned by psychological predetermination. No experience can change their property of continuity—among others which do not interest us here. Try as hard as you like to imagine discontinuous time or space; you will find it as impossible as to visualize time of two dimensions, or space of four dimensions. Furthermore, there is no physical experience that forces us to abandon our conception of time and space. The theory of relativity simply suggests a change in the units employed to measure them, and a rearrangement of the order in which observable phenomena appear in time and space. That is perfectly compatible with our views. So is the suggestion that so-called "physical things" may not have a continued and identifiable existence in time and space. There is nothing in the concepts of time and space that would prescribe the behavior of phenomena with regard to them, although

both carry with them inseparably the property of continuity. In fact, the very statement of the discontinuous existence of things implies a continuous background for judgment. Thus we find that this seemingly paradoxical and monstrous consequence of Heisenberg's theory is not to be rejected on general grounds, but is consistent with the requirements of a good explanation. However, this is no proof for its correctness, nor a reason why it should be advocated, and I should say that if the suggestion has no stronger a support than Heisenberg's much too famous uncertainty argument, it is interesting only because of its novelty.

A treatment of the nature of physical explanation is incomplete unless it deals with the function of probability and statistics. It is understood that until very recently the theory of statistics has been a powerful method of describing aggregate phenomena, used discriminately by careful investigators with a proper sense of its limitations. Of course Boltzmann introduced the postulate that the elementary motions of molecules are chaotic in order to provide a basis for the application of probabilities. But he regarded his assumption as an approximation which derived its validity from the existence of a large number of molecules. With this understanding, statistical concepts are symbols of description rather than of explanation, a concession of ignorance rather than an assertion of definite knowledge. Yet, why should probability relations not be incorporated in a system of explanation? There is nothing in our requirements about explanatory symbols that would brand them as objectionable. Clearly, it is the assumption on which they are founded. If the postulate of chaotic motion be detached from the condition that there are very many molecules, that is, if causal relations controlling the motion of a

single molecule are dissolved and annihilated, then we are violating the principle of consistency of nature upon which the possibility of explanation rests. To be quite accurate, I should have said that this violation is incompatible with the theory of explanation which I have attempted to present. Whether or not an alternate theory can be constructed, independent of the axiom that nature is consistent, I am unable to say at present. This can, however, not be denied on *a priori* grounds.

With the reader's kind indulgence I shall conclude this discussion by a few remarks on the relation between explanation and reality. Reality has not yet been defined in this essay, nor anywhere, as far as I am aware, in a satisfactory manner. Every attempted definition makes an assumption about nature that is unwarranted by sufficient evidence. But we are still in the process of learning about nature, and that process will continue so long as there remains a science. Hence it will forever be hopeless to determine reality in terms of an existing nature of which we have no cognition. Moreover, the concept of reality has nothing absolute and unconditional about it, neither in its logical nor in its philological implications. A study of the various, sometimes conflicting, notions which the word reality has been used to convey throughout the recorded history of the race, is exceedingly interesting. It would show convincingly that the meaning of the term is variable, subject to arbitrary fixation. And this fixation must be performed with closest reference to our knowledge of nature. Since we can not define the latter in terms of the former, the argument seems cogent to me that reality must be the best system of explanation, the ultimately satisfactory one. Thus reality has been taken from the dusty store-house of logical categories and removed to the lofty heights of

ideals. Its substance has vanished before our eyes and reappeared as the light of a shining star, guiding us in scientific labors. Reality does not exist as a discoverable something in the phenomena of nature, called into being by the single act of a designing spirit of the universe. We do not *discover* but continually create it in our efforts of designing *the* ultimately satisfactory system of explanation.

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WHAT IS A QUESTION?

"WHAT is a question?" is a question which seems to have been almost totally ignored by logicians. The problem is, however, about as important for rational thought as the more common inquiry into the nature of propositions, assertions, or judgments. And if the former inquiry does, in its claim to significance, presuppose a solution, so too does the answer to the latter. That is to say, in order to answer the former question we must assume that it is a question, just as we must assume that any real definition of a proposition is a proposition.

In neither case, however, does this consideration involve a vicious regress. And if our question can be answered, the ultimate value of such a solution to philosophy must be considerable. For it is obvious that many apparent questions lack significance, that for want of recognizable criteria of interrogatory significance much philosophical discussion consists of a useless attempt to answer meaningless questions, that a good deal of superficial and unjustified support is given to the skeptical or inquiring attitude as opposed to the dogmatic because of a failure to realize the intellectual responsibilities determined by the logical presuppositions of significant questions, and that a cloud is thrown across many philosophical problems by a failure to analyze the general relation of a question to an answer.

I

A question is not, as some logicians imply by their treatment or lack of treatment, simply a psychological provocation, on a par with pin-pricks and miracles, to the formation of assertions. If it is true that questions are valuable because they lead to judgments, it may also be true that judgments are valuable because they lead to inquiries. Perhaps it is an undue pre-occupation with rats in mazes which leads some of us to assume that thought is valuable only as a method of getting out of difficulties, a "means of converting the dubious into the assured, and the incomplete into the determinate."[1] In our intellectual mazes there is wonder and adventure more thrilling, frequently, than the cheese which lies outside the cage. Those who have formulated the world's problems have more often deserved the name "philosopher" than those who have settled them. There is thus a certain superficiality in the ethics which regards thinking as wholly pragmatic and concludes that the question is the beginning of thought, important only as an instrument for attaining the end of thought, the judgment. Some such valuation seems to be at the basis of the logician's exclusive concern with propositions and his indifference to questions.

But we trespass upon the domains of psychology and ethics. Whatever the reason for its neglect, and whatever the value of its cultivation, there is, in the analysis of the question, a virgin field for logical exploration.

The question has usually been described as a request for information.[2] But while it is true that we generally

[1] Dewey, *"Experience and Nature,"* p. 67. Professor Dewey, of course, is not responsible for the anti-intellectualist morals which have been drawn from this doctrine.

[2] This is the explanation offered in Clauberg and Dubislaw: *Systematisches Wörterbuch der Philosophie;* Eisler: *Hand-wörterbuch der Philosophie;* and Eisler: *Wörterbuch der philosophischen Begriffe."* In the last named work the names of Bolzano, Natorp, Jerusalem, Wundt, and many others are adduced in favor of this position. The definition of *"Frage"* given by Lipps is characteristic: *"der Wunsch zu einem Urteil zu kommen."*

ask questions in order to get information, it is also true that certain questions (e. g. rhetorical questions) are presented with no intention of receiving answers. Other questions (e. g. "What is the largest number?") have no answers. And, finally, the idea that a question is a request for information does not in the least explain the nature of questions. If I ask who discovered America, I am none the wiser as to what I have done when told that I have requested information. What information? Why, of course, information as to *who discovered America.* In short, our desire to receive an answer when we ask a question is, like our desire to be believed when we assert a proposition, neither universally present nor in any way constitutive of the meaning or content of what we ask or assert. What is it, we must go on to inquire, that we want believed? What is it that we want answered?

On another common view, a question is simply an ambiguous assertion. But clearly, Spencer's definition of evolution, however ambiguous, is not a question. If an ambiguous assertion is a sentence which has more than one meaning, then a question is not such an assertion. For many questions have only one meaning, and, on the other hand, many sentences that have several meanings are not interrogative. If by "ambiguous assertion" is meant some kind of proposition, then no such assertion can be a question, since every proposition is either true or false and no question is true or false.

Finally, a question, like a proposition, is not simply a psychic event or a physical object,—it is a logical entity. Marks on paper, sounds in the air, activities of brains, and incidents in psychical history cannot be true or false or have true or false answers. They are objects or events, to be evaluated in terms not of logic but of ethics. It is

only with the meaning of these signs, the content of these thoughts, that logic is concerned. Unfortunately we have not two words to denote these different entities. Logicians distinguish between a statement (or judgment or declarative sentence) and a proposition (or assertion). The one is a human act or a symbol; the other, a logical entity, a meaning. But the word *question* is used indiscriminately to refer on the one hand to the act of questioning or the verbal symbol, and on the other hand to the content of the thought, the meaning that is entrusted to and communicated by a conventional language sign. From the confusion of these two entities no adequate consideration of our problem can emerge. We shall therefore use the word *question* to denote exclusively the logical entity, and refer to the act or symbol embodying this as an interrogation or interrogative sentence. It is with the former entity alone that we are directly concerned. And with these qualifications we return to our original problem, "What is a question?"

II

A question, it is submitted, is simply a propositional function (or propositional form). "What is the sum of 3 and 5?" seems to be identical in logical content with "$x = 3 + 5$." Whatever difference appears between the two phrases seems to reside merely in the psychological connotations commonly adhering to the different styles of expression. That is to say, we generally want an answer when we ask a question, although we frequently put a propositional function without any demand that its values be supplied. But this matter of compulsive flavor, in which our two expressions may find a shadowy distinction, does not go to the logical content of either.

As a logical entity the question is the clear embodiment of the characters by which the propositional function has been defined. It is neither true nor false, while its values (answers) are true or false. It is of the form of the proposition, yet differing from the latter by the substitution of a variable for some constant. *Who, which, what, when, where, why,* etc. are the variables of every-day speech.

As in mathematical logic, these variables have a dual use. As "real variables" they appear in the rôle of interrogative pronouns or adjectives. As "apparent variables" they are termed relative pronouns or adjectives.[3] Thus the existential affirmative, $(\exists x).\phi x$, is simply "There is something *which has the property* ϕ." Just as the ϕx, taken by itself, is a real variable, a propositional function, so the relative clause in our translation is, when independent of the main clause, a question. The universal affirmative, $(x).\phi x$, or "Everything is a thing *which has the property* ϕ," may be similarly analyzed. So we may go through the uses of the propositional function as an independent and as a dependent entity, and find exact, though frequently cumbersome, translations in the questions of common speech.

On this analysis, an answer to a question must be simply a proposition which is a value of the given propositional function (or, by ellipsis, a constant term which is a value of the variable in this function,—the difference between these two views is unimportant for our present inquiry).[4]

[3] For example, in "What is red?" *what* appears as a real variable and produces a question, a propositional function. In "What is red is colored," we find an apparent variable (*what* equals *whatever*), producing a universal proposition. The two uses of *what* are more easily confused in mathematics and logic than in ordinary conversation.

[4] The word *information* is, I think, very significant. Even in its purely psychological aspect, indetermination or doubt is not, as is often maintained, a wavering between different certainties, but the grasping of an incomplete form, a variable.

A true answer is simply a value that is true. We shall defer further consideration of the relation of questions and answers to another section.

There are some questions, finally, that do not possess in so obvious a form as those thus far considered the character of the propositional function. Such in general are the questions that contain no interrogative pronoun or adjective (e.g., "Is Caesar dead?", "Caesar is dead?"). In what way, we may ask, does the logical entity denoted by these sentences differ from that which is denoted by the related affirmation "Caesar is dead." Again we must dismiss from consideration such psychological data as our desire for information in the former case and our belief in the latter. Considering simply the content of our thoughts, we find, I think, that in the former case there is no assertion, but simply the ascription to a specified (but unasserted) proposition of an undetermined truth-value. Whereas in the questions previously considered, a specified term was the variable and was denoted by a special interrogative word, now the variable is the truth-value, or validity, of a proposition. There is obviously no logical reason why there should not be an English word representing the variable whose values are *is* and *is not*. Were that the case, we should represent our question as "Caesar blankety-blank dead?"[5] But this fortunate grammatical omission in the English language should not obscure the fact that questions of this sort are essentially similar to those already considered,—that they are in fact a sub-class of propositional functions, in which a variable taking the two values truth and falsity, or fact and not-fact (com-

[5] This "blankety-blank" is what Professor Sheffer calls the "validational variable." See the article "Notational Relativity" in the *Proceedings of the Sixth International Congress of Philosophy*. My indebtedness to Professor Sheffer at other points in this paper will be apparent to those who are familiar with his work.

monly represented, in the answer, by the words *yes* and *no*) appear. The identification of questions with propositional functions is thus complete.

III

This process of translation is not entirely of one-sided value. For if it offers us a clue to the logical nature of the question and, as we shall later try to show, to the analysis of interrogatory significance, it provides, on the other hand, an intelligible and clarifying interpretation of the propositional function.

To interpret "x is mortal," to mean, as Russell suggests, "anything is mortal" is either a confusing misuse of terms or a downright fallacy. "Anything is mortal" in common speech means simply "everything (distributively) is mortal." In negative statements, *anything* frequently means *something*. Thus "I do not think that there is anything which is mortal" means exactly the same thing as "I do not think that there is something which is mortal." "Everything is mortal," and "something is mortal" are both propositions, and neither is what we mean by "x is mortal," which is not a proposition at all.

This misinterpretation of the propositional function cannot affect the validity of inferences which formal logic makes concerning the entity. It has, however, seriously clouded the philosophical implications of an exceedingly fertile concept. For when ϕx is interpreted as "anything has the property ϕ," it is only natural to confuse the proposition which these words commonly denote with the propositional function they are meant to denote.

Thus in the *Principles of Mathematics,* Russell refers to ϕx sometimes as a proposition (p. 92, §90), and at other times as a propositional function (p. 13, §13; p. 92, §90;

p. 93, §92). In regard to the meaning of $\phi x \supset \psi x$, (ϕx implies ψx) the confusion is dizzying. On pages 38 (§41) and 91 (§89) this is a proposition. On pages 38 (§42), 87 (§83), 89 (§86), and 92 (§89), it is a propositional function.[6]

In the *Principia Mathematica* (first edition), an attempt is made to remedy this confusion. A new concept, intermediate between the propositional function and its values, is introduced. This is the so-called ambiguous denotation of a value. ϕx is no longer a propositional function, nor is it a particular value of a function. It denotes ambiguously a specific value, but since we do not know whether it refers to ϕa, to ϕb, or to ϕc, we can assert it when and only when every such value is true. ϕx, then, is equivalent to $(x).\phi x$, but not identical with it.

This doctrine, it appears, is based upon a fundamental confusion between the symbol and the thing symbolized. The propositional function is not an ambiguous or indeterminate symbol. It is a symbol of an objectively indeterminate (i. e. variable) thing. But this new ϕx is not a definite symbol of an indeterminate entity. It is itself an ambiguous symbol. As such it has no place in logic. Logic does not deal with the symbols in which it is expressed. Expressions like "He is mortal," (where the "he" has no visible reference), and "Interest comes," (where it is not known whether the sentence is in English or Latin), as

[6] When we get rid of the word *any* and translate propositional functions as questions, the grounds for this confusion vanish. $(x).\phi x \supset \psi x$, or "Everything which has the property ϕ is a thing which has the property ψ," is a universal proposition, which can no longer be confused with the propositional function, $\phi x \supset \psi x$, or "What has the property ψ if it has the property ϕ?" Russell's argument that the latter expresion must contain two variables—thus meaning, "What has the property ψ if what has the property ϕ?", a double question—is based upon a confusion of logic and typography. The fact that the symbol x appears twice in the expression does not prevent the sign from meaning the same thing in both appearances. We might, if the type-setter were willing, so arrange the expression that the x appeared only once. A similar confusion, it may be noted, appears in the primitive proposition *1.4 of the *Principia Mathematica.*

well as most typographical errors and partially inaudible remarks, are strictly cognate with the "assertion of an ambiguous member" of the values of a propositional function. As symbols they are no part of the subject-matter of logic. The things they symbolize in their various interpretations are either propositions or propositional functions, and there is no intermediate logical entity to be described by the word "any."

In the second edition of the *Principia Mathematica,* Russell and Whitehead show that what was previously asserted in the form of ϕx, (where ϕ is a constant), may be asserted in the form of the universal proposition, $(x).\phi x$,[7] and state somewhat cryptically that the "assertion of a propositional function" is no longer needed. We may infer, I think, that the reason it is no longer needed is that it does not exist. One cannot "assert" a question.

We shall not pursue any further the fatal errors consequent upon the mistranslation of the propositional function in terms of "any,"—although an interesting chapter might be written on the manner in which the ambiguous word "any" has been used to bridge the gap between universal and existential propositions.[8] It is sufficiently evident from the preceding considerations that a propositional function cannot be translated by a universal or existential proposition (through the use of the word "any"), or by any expression that ambiguously refers to one proposition

[7] In ordinary mathematics a similar ambiguity of symbolism demands a similar remedy. "$\sin^2x+\cos^2x=1$" is generally construed as a universal proposition, and "$\sin^2x=1$" as a question. The former expression may be interpreted, however, as a question, and the latter as a proposition. In that case the question is indeterminate and the proposition false. In order to get rid of this ambiguity it is necessary to distinguish between the two uses of x (or, in translation, between the interrogative and the relative pronoun). This can be done by writing universal propositions consistently in the form $(x).\phi x$,—in this case, $(x).\sin^2x+\cos^2x=1$, and propositional functions in the form ϕx. A change in the form of the equality sign sometimes serves the same purpose.

[8] *Cf. Principia Mathematica,* pp. 19-20 (in both editions).

or another. The difficulties and errors attendant upon any of these interpretations are swept away instantly when it is recognized that the propositional function is identical with the question. "x is mortal," stripped of its text-book disguise, is simply the familiar question, "What is mortal?" On the other hand, it is only by the converse translation that we recognize the logical content of the interrogation, the instrument and the resultant of judgment, the unfolder alike of destiny and human ignorance.

IV

The foregoing considerations may be viewed as defining question and answer in the widest sense of the terms. Thus every propositional function is a question, although it may be indeterminate or insignificant, and every value of such a function is an answer, although it may be false. This terminology does not constitute an untoward strain of language, since we do commonly apply these adjectives to certain questions and answers. And in any case, it offers a clear verbal framework for the essential problem that remains to be considered. What questions are significant, and what answers are correct?

By a significant question, I mean a question to which some proposition is *the* true answer. Two things are thus demanded for interrogatory significance. In the first place, there must be at least one true proposition that is a value of the given propositional function, and in the second place, there must be not more than one such proposition. Questions which violate the former condition may be appropriately called *invalid*. Thus "What cat has eight lives?"[9], "Who discovered America in 1491?", and

[9] It might be supposed that "No cat has eight lives" is a correct answer to this question. But although this may be a very appropriate retort to an invalid question, it is entirely different in form from a real answer, being a negative universal, while the values of the propositional function advanced are all particulars. An oversight at this point tempts the inference that since one cat has one more life than no cat, one cat has nine lives.

"$x \cdot o = 1$" are typical examples of invalid questions. We may, without committing any logical fallacy, ask such questions as: "When did you stop beating your wife?", "What is the highest good?", "Where is the mind?", and "What are the ultimate simples of sense-experience?" But we do fall into error when we assume, (as we usually do when we ask questions), that such questions must have true answers, and ignore the fact that to justify the validity of these questions it is necessary to show that the person addressed has stopped beating his wife, that there is a highest good, that the mind exists in space, that there are elements of sense-experience which are ultimately simple, etc.

Every propositional function lays down a range of significance determined by the possible values of the variable term, and an inner range of truth further determined by the constant terms of the expression. Thus the presumption of validity in a question is an assumption that this latter range (and therefore the former range as well) contains at least one member. Such an assumption will be true or false. When false, any answer to the question must be incorrect. The chief usefulness of questions (apart from riddles) arises from the fact that we can sometimes know that such a value exists without knowing what it is.

The second condition of what we have called a significant question is that it have not more than one true answer. Questions which violate this requirement may be called indeterminate. Thus, "Who did what when?", and "$1^x = 1$," are indeterminate and therefore non-significant (in our defined sense of that word,—we do not mean to imply that invalid or indeterminate questions have no meaning). To such questions we may indeed give true

answers, but we can never give *the* true answer to any of them. Thus in claiming significance or simply determinateness for a given question,—and we do this whenever we attempt to show that one answer is incorrect by demonstrating that a materially different answer is correct,—we are under the responsibility of showing that not more than one true proposition is a value of our propositional function. The relevance of this principle to philosophical discussion is obvious. "What is the first mover?" in a world where rest and motion are relative to variable coordinates, is the type of a great class of questions which lead inevitably to error when they are regarded as determinate. The fundamental question of ethics, "What is the good?" has regularly been treated as if it were (abstractly) determinate. Thus the more basic question of whether *good* is a constant or a variable (similar to *mine*) is never clearly faced and always unconsciously answered.

At this point a distinction of crucial importance must be made between questions that are indeterminate and those that are ambiguous, applying the latter predicate to questions which have no uniquely determined meaning. An indeterminate question we have seen to be a definitely denoted propositional function which has more than one true value. But an ambiguous question is not, in the logical sense, a question at all. It is rather a group of questions, or, more accurately, an ambiguous symbol, a verbal matrix from which various questions may be derived. In asking whether certain things are real or practical or right, I may have in mind something quite different from what another person understands by the words. What would constitute a correct answer to the question in my mind may be a false answer or no answer at all to the question in my neighbor's. But both of these questions

may be determinate and significant. Ambiguity, then, is something which attaches not to the idea which a set of words suggests, (and it is with the analysis of such ideas that we are concerned), but to the set of words itself in so far as it suggests various meanings.

That words and sentences, declarative or interrogative, do convey different meanings to different people and even to the same person in varying circumstances is too obvious a fact to be labored, yet the ignoring of this fact is perhaps the most fertile source of philosophical and non-philosophical argument. Bertrand Russell somewhere says that no two philosophers ever understand each other. If one may confess to an understanding of that remark, it appears to be very near the truth. Certainly we shall never bridge the chasms about a human soul with our primitive marks and noises, but if there is to be any rational intercourse between man and man, we must somehow approach the ideal of unambiguous speech. And to do this we must remember that the ideal is beyond the language that pursues it.

Ambiguity is as prevalent and as dangerous in our interrogations as in our statements, but the problems which it raises in this connection are problems of thought and human intercourse in general, and as such are irrelevant to a study of the logical nature of a question, except in so far as they help to explain what we are not talking about.[10] If, as Professor Whitehead hopes, we shall find real propositions in the kingdom of heaven, there too shall we find real questions. But it is the divine task of the

[10] The distinction between indeterminateness and ambiguity is paralleled by the more obvious distinction between invalidity and meaninglessness. A symbol which has no meaning,—e. g., Wittgenstein's creation, "Is the Good more or less identical than the Beautiful?"—is not, in the logical sense, a question. But a question may have no true answer. In other words, the predicates meaningful, meaningless, ambiguous, and unambiguous refer to interrogations and interrogative sentences, but not to questions. Meanings do not *have* meaning.

logician to examine these ideal entities that we may better discern meaning and direction in the world of human thought. The significant question is, like every object of reason, an abstraction from actual experience.

The possibilities of analysis and classification which unfold with the realization that questions are variables whose values are answers go far beyond this problem of interrogatory significance. In particular some light is thrown upon the nature of complexity in questions. In the days when logic was thought of as a branch of ethics, the text-book writers used to tell us that we ought to phrase our inquiries so as to ask one question at a time. By this they meant, sometimes, that we should avoid ambiguous speech, at other times, that we should not ask questions in which assumptions are already implicit. The former condition is perhaps psychologically unattainable, although there are important differences of degree in its approximation. The latter is logically impossible, since, as we have seen, although no question as such makes an assumption, every question in so far as it demands *a* or *the* true answer does make definite assumptions.

But there is a third more important dimension of complexity in questions, namely that which relates to the number of variables in a given propositional function. In common speech, questions containing more than one variable are usually indeterminate, and such questions as "Who's who?", "What's what?", etc., are frequently convenient precisely because of their wide range of true answers. But there is no logical correlation between complexity and determinateness or validity. We may have double questions (i. e. propositional functions containing two variables) which are valid and determinate,—e. g., "Who discovered America in what year?", "Did Caesar

kill Brutus or did Brutus kill Caesar?" The same is true of questions of higher degree of complexity. In the analysis of complex questions and of their relations to simple questions and to answers, many points of interest are raised. Their discussion, however, would take us beyond the bounds of space and subject-matter we have set.

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NEW YORK CITY.

A THEORY OF MATTER

MOST scientists take it for granted that they know what matter is. It is very easy to show that they are mistaken, for they are not at all aware that there are other means than science to study matter. Whoever wishes really to understand the nature of matter and of the material world must take into consideration and compare all the possible kinds of human experience which reveal something about matter and the relation between matter and spirit. Let us try to ascertain how many different human activities serve that purpose.

1. *Science.* The particular sciences are not all on a level, but form an ascending scale of efforts towards a fuller knowledge of different aspects of the material world. The oldest of the natural sciences is astronomy. Observation of the stars shows us large bodies moving in unlimited space. This primitive image of shining points in movement is an object of sight, and we commonly conceive the whole matter of the visible universe as a system of shining points in movement. This is the original type of the whole atomic conception which claims to explain what is happening in the universe by the movements of very small bodies in space. Between the original matter of the astronomer and the matter of the physicist there is no other difference than that of dimensions.

The astronomer looks at the whole universe accessible to our view, the physicist divides every minutest quantity

of visible matter into invisible particles. The stars are visible, the molecules are invisible; and the astronomer perceives matter only by sight, while the physicist is concerned also with sensations different from those of sight, namely sounds, and electric and magnetic phenomena. Thus it would seem that the matter of the physicist ought to be more differentiated and more complicated than that of the astronomer, physical science being a more advanced stage in the scientific investigation of matter than astronomy, and further from the immediate sensation.

But strangely enough the physicist accepts the visual astronomic conception of matter integrally and makes no use whatever of his other sensations. He transforms sounds into waves of molecules in movement, and treats in the same way electricity and magnetism. His conception of matter as bodies in movement is essentially the same as that of the astronomer, and the increased variety of the quality of sensations changes nothing in the visual scheme.

The molecules are invisible, but they move in the same space as the stars, and the physicist does not add anything from his particular observation of matter to the original conception of the astronomer. Astronomers do not see the movement which they imagine, as this movement is too slow. They notice a change in position and they infer that the stars are moving. The physicist similarly does not see the movement of his molecules. He notices a rise in temperature and infers that it is produced by accelerated movement. The physicist does not measure temperature by his tactile impressions. He looks at the scale of the thermometer and he draws inferences from his visual impressions. He reduces everything he sees, hears or touches to the astronomic pattern of bodies moving in space. No amount of physical experience obtained by other senses

changes anything in the simple visual scheme of the movement of stars.

We notice the same conservatism in chemistry. The chemist receives from the matter he investigates sensations of smell and taste, but he makes no use of these sensations in the conception he has of matter. That conception remains the same in chemistry, physics and astronomy. Chemists, as well as physicists, use different senses in their perception of material processes, but they limit themselves to the visual conception of the movement of a body in space when they endeavor to explain what they perceive by other senses.

The imagined movements of molecules, atoms or electrons are essentially of the same quality as the imagined movements of the stars and planets, the only difference being a reduction of size. If we say that a taste or a smell is the result of atomic action on our senses, we simplify and unify the variety of sensations, reducing them all to the visual image of the movement of a body of definite size with a certain velocity through space.[1]

Thus it is evident that we utilise for our conception of matter only a small part of our sensations, those of sight. But our sight is by no means the most important of those senses by which we perceive reality. History tells of many sages who were blind, but no deaf man ever became famous for wisdom. Hearing is more important than sight for the knowledge of reality, because it establishes mutual relations between men and enables us to compare our sensations and thoughts with those of others. Acoustics form an important part of physical science and the study of

[1] An attempt to explain the reality of atoms by their inward life was made by the author in pp. 93-99 of his *World of Souls* (George Allen and Unwin, 1924). This conception of a spiritual atomism or monadologism belongs to Leibniz, but it is made superfluous by the new theory of matter resulting from a systematic survey of all the ways leading to a final knowledge of matter.

waves of sound makes it easier to study the waves of electricity and light. Notwithstanding that, sounds count for nothing in the atomic conception of matter, not any more than do sensations of taste, smell or touch.

If we wish to judge the quality of a fruit, the best method of investigation is to eat it, and no explanation of the unknown atomic movements which are supposed to produce its taste would give us as much real knowledge of that fruit as mastication, deglutition and digestion. Nevertheless these experiences are ignored as irrelevant, and naturalists believe themselves to have sufficiently investigated a fruit when they have analyzed its anatomic structure and chemical composition and traced its origin. Here materialism is inconsistent, for if matter is the object of sensations, we ought to utilise all our sensations and not only our visual perceptions in order to understand matter.

Above physics and chemistry stands the domain of biology. The biologist, like the chemist, receives many various sensations from the objects of his study, and, like the chemist, he tries to reduce everything to visual images, imagining them to afford a sufficient explanation. His favorite instrument is the microscope. But the living cell differs from the chemical atom and from the physical molecule, as it shows us a continuous flow of assimilation and excretion, unlike concrete bodies in movement.

Organic life produces sensations of taste, smell and touch, not utilized for the metaphysical conception of atomism. Biological realities are translated into visual schemes of the astronomical model: the astronomical tradition has dominated not only physics and chemistry, but biology as well. It is always the image of very small bodies in eternal movement by means of which biological facts have been represented. The taste of a fruit, the scent of a flower, the tactile impression of a caressed animal are considered

to be subjective illusions and the only reality is held to be the movement of atoms or electrons, similar to that which was first ascribed to the stars, though neither the one nor the other has ever been seen.

In vain does biology reveal to us more and more complex mysteries of organic life. It is only at the last stage of the study of organisms, in mediumistic materialisations, observed by metapsychists, that scientists have been obliged to recognize that these manifestations differ from the movements of the stars, molecules, atoms or electrons not only quantitatively, but qualitatively also. A materialisation is not the movement of a body large or small.

Here at last we hit on something new that can no longer be explained in the old way. Suddenly there appears a hand, a face, or a complete organism with a beating heart, and as suddenly they disappear. This teaches us much more about the intimate nature of matter[2] than the whole of astronomy, physics, chemistry and biology, because these manifestations subserve a purpose which is not a material necessity in the same sense as astronomical, physical or chemical necessities.

Matter acts here on all our senses, but under the influence of an immaterial force. Matter here appears as an instrument of the materialising spirit for the expression of some intellectual or spiritual contents. No such content is immediately revealed in the movement of the stars, nor in the supposed movements of atoms and electrons. Looking at bodies in movement we are liable to forget that they may depend on some spirit, we take their movements as something that dominates our spirit, and therefore as a reality independent from the spirit. Mediumistic matter is obedient to the spirit, and could by no means cause or

[2] The theory of materialization has been dealt with by the author in his recent work *Preexistence and Reincarnation*, published by George Allen and Unwin, London, 1928.

explain the activities of the spirit, as has been sometimes supposed of the movements of invisible bodies in our brain and nerves.

The study of astronomy, of physics, of chemistry relates to inorganic matter; biology extends the field of inquiry to the organic world, but still remains under the spell of the astronomical tradition, and we have had to wait for metapsychics in order to make a new departure in our study of matter. These experiments furnish the elements of a conception different from the old atomism. The matter of mediumistic materialisations cannot be explained by movement of atoms after the pattern of the stars. It is matter of a fluid continuity, not to be seized and fixed by sight, as it mocks the senses, suddenly appearing and disappearing in obedience to an immaterial power.

The star, the molecule, the atom, even the electron are permanent centers which appear to be the elements of manifold combinations. This gross matter dominates and enslaves the spirit, while mediumistic matter is created by the spirit, is subordinate and obedient to will power. Thus we see that the succession of sciences leads us beyond materialism, and brings us to the conclusion that matter is not a real substance, but something relative and subordinate, dependent on a higher, that is, a spiritual reality.

There were indications of this conclusion as soon as we went beyond the domain of sight, which is the only sense used by the astronomer. Sounds are already something more flexible and fluid than stars or stones. Though for a particular purpose translated into waves, the sounds had their own invisible reality of rhythm and measure, very different from the fixity of a revolving star. The concept of a wave, when applied to light and electricity was so inadequate, that for a long time two competing theories remained in the field.

Electric and magnetic phenomena have shown us a form of matter increasingly immaterial in comparison with the blocks of granite of the geologist. The qualitative transformations due to chemical reaction were never really accounted for by atomism. The process of combustion, which changes hydrogen and oxygen into water, is a qualitative transformation, a kind of materialization, akin to that of the elusive materializations produced by mediums. In both cases something suddenly appears to the senses out of the invisible.

Biological phenomena were long acknowledged to contain more than physics and chemistry could account for and vitalism tried in vain to satisfy our intellectual need of understanding the difference between life and death, between organic and inorganic matter. It was reserved for the highest stage of biological research, for metapsychics, to emancipate human intelligence from the naive representation of bodies in movement as explaining everything material. We have learned that bodies may be created by the spirit and that they disappear as soon as the spirit does not sustain them.

This is the most important step since the time of Democritus in the understanding of matter, and leaves us to look for other than scientific means in probing the relation of mind to matter. Science utilizes perceptions built out of sensations. These sensations are independent of our will, being given to us as something from without, and on which we have no influence except through experiments, kept within a narrow range.

Thus scientific research deals with matter as it acts on man and only exceptionally with the action of man on matter, or the modifications which ensue. There is another vast domain beyond the range of science, in which from the beginning man dominates and shapes matter. Herein

man acts in a totally different way, for instead of observing what already exists he attempts by his will to transform reality. He begins where the biologist ends, and his activity raises mankind to a higher level of spiritual achievement than any at which science ever aimed. That strange magician who starts his career with those materializations which for the scientists are the latest and most puzzling discovery, is the artist. His purpose is to impress on external matter some form pre-existent in his spirit.

2. *Art.* Let us compare the artist and the scientist when confronted with a block of marble. The scientist will explain the geological origin of the stone, and when he has dissolved it in hydrochloric acid will prove that it consists of carbonate of lime. The artist will carve a statue out of the shapeless mass. Which of the two will learn more about the intimate nature of marble, the chemist who destroys it and produces invisible carbonic acid out of it, or the sculptor who reveals a form hidden in it and animates the stone by giving it the shape that corresponds to the latent possibilities of that kind of matter? Marble becomes alive in the hand of the sculptor, and it is destroyed by the chemist.

Which is more important for the human intelligence: to learn of the thousand shapes that can be given to marble, or of the few chemical elements contained in it? What shall we say of the chemist who, being offered a delightful peach, instead of eating it, will submit it to chemical analysis? Which of these two men is nearer the truth, the chemist who despises the gourmet for one kind of ignorance, or the gourmet who despises the chemist for another kind of ignorance? If we wish to understand the nature of marble, is it better to try all the shapes that it can take and learn of the limitations imposed on the artist by the quality of that matter, to compare the same shape worked out in

different materials and to decide what material best suits a proposed form—or shall we rather reduce to a chemical formula or to a geological hypothesis the quality of that stone without asking to what use man might put it?

It is evident that in relation to matter of a particular kind, such as marble, the sculptor is convinced of his superiority over the chemist or the geologist. It is the use of matter that determines its quality for man. We esteem the quality of a peach by the taste, the quality of tea or wine by drinking it, the quality of marble, bronze, ivory or wood by admiring the artistic forms which such different materials can take.

In art, as in science, there is a succession of degrees or stages which lead to a deeper knowledge of matter. But art is soon emancipated from the predominance of one sense over others, and therefore we may recognize the superiority of art over science in the study of matter. Sculpture and architecture are the first stages of art, corresponding to astronomy among the sciences, but using in addition to sight the senses of touch and muscular effort. They produce and give permanence to visible forms, chiefly in imitation of living organisms. There is something wonderfully impressive in the permanence of buildings shaped by architects long ago and outliving their makers by centuries.

The art of dance forms the expression of a living body in movement: Valery's Platonic dialogue *L'âme et la danse* (published in 1924, almost simultaneously with the *Dance of life* of Havelock Ellis) demonstrates the sublime dignity of that old art, which has often degenerated into a frivolous and sometimes obscene diversion. Towards the end of the last *Ennead* of Plotinus (VI, book 9, chap. 9) we read a wonderful passage in which the highest perfection of Life is represented as a Divine dance. These three

arts, sculpture, architecture and dance, show us matter in permanent shapes either static or dynamic, at rest or in movement.

The artistic dance of a beautiful body, inspired by a soul loving measure and harmony, is an experience in the domain of art which corresponds to the astronomer's contemplation of the slow dance of the stars, with this difference, that the astronomer has no influence at all on the movements he observes. But whether we look at the stars or at the dance of a woman or a group of dancers, we see bodies in movement. We see the movement in dance and we guess it only in the stars or atoms. If a body in movement is to become a pattern of reality, it is rather the famous and fair Athikte in Valery's poem than any star that can reveal to us what a body in movement really is. Her movements are quicker, more unexpected and variable than the movements of the stars, for she manifests movement not only in change of place, as do the atom and the star, but also in the change of her expression, her attitude and gesture, which of course does not happen in the case of atoms or stars.

The true body in movement is not a luminous point, it is a changing artistic shape like that of Athikte, a poet's immortal creation. We should come nearer to absolute reality if, instead of the whirl of dust sung by Lucretius, we took as the original pattern of bodies in movement a well trained group of beautiful dancers, moving rhythmically in tune with a skilled orchestra. We should then notice the wonderful relation between the slight movements of the conductor of the orchestra, the more extended movements of the musicians, and those most expressive of the dancers. In the whirl of atoms or stars we miss the conductor who inspires the rhythm, and everything appears to be mere chance. In artistic dance there is a

revelation of the nature of matter which surpasses all the teachings of natural science.

A fourth art leads us still further from the primitive impression of gross matter, though it does not appear to imply any movement. The painter reduces to two dimensions on a plane surface the reality of three-dimensional objects, and though the picture itself does not move, the objects are often represented as moving, and then the painter's art is able to produce this illusion of movement.

This transformation subtilizes matter in a strange way, amounting to a partial dematerialization of the shapes presented to our sight. These shapes are animated by colors, which give them the appearance of movement and life to an extent that is scarcely possible in sculpture and quite impossible in architecture. The painter uses a variety of colors and shades for his work, while the sculptor and the architect have usually a very limited variety of materials for their use.

Thus it is true that the painter, when he tries the effect of a particular pigment, as for instance cinnabar, grasps more completely its true nature than the chemist who decomposes cinnabar into sulphur and mercury. What the painter knows about cinnabar is much more important to the human mind than what the chemist discovers. The chemist speaks of the elements hidden in cinnabar, but cinnabar decomposed is no longer cinnabar. The painter shows what various appearances cinnabar can produce in relation to other pigments, and thus he manifests cinnabar as it really is, not split into parts which bear no resemblance to the whole.

The immediate contrast of many colors in a picture is much more complicated than the form given to a single kind of matter by the sculptor, or to a few kinds of matter by the architect. Painting represents all possible shapes

imaginable by sculpture, but adds color and intensifies the impression of movement, so that it helps us to seize the most intimate essence of matter, better than sculpture, architecture or dance, if we take dance only as it appears to onlookers, disregarding the muscular sensations of the dancers which belong to a much more advanced stage of our experience of matter.

Painting also represents wider horizons, a larger part of the material universe than either sculpture or architecture can display. On a small surface many miles of landscape may be concentrated by the painter, including many buildings, perhaps even master-pieces of architecture and sculpture. This power of the painter to condense space raises him above the sculptor, the architect or the dancer, and we conclude that the art of painting is in that respect superior to the three preceding arts, as physics and chemistry are superior to astronomy in the knowledge they give of matter. The process of lifting pictorial art above the function of copying nature so that it may reveal colors that are rarely seen and shapes that never existed, leads us still further in the study of matter, to the utmost limit of what sight can reveal about material possibilities.

Above all the plastic arts there is an art which liberates us totally from the bondage to sight and deals with invisible and impalpable matter of a peculiar kind. This is the art of music. Sound is material, being the object of sensation. The production of sounds in a certain order, according to the will of the musician, makes use of matter in a manner different from the plastic arts and opens to us the knowledge of a material world without shape or color, yet real as statues or dancers.

Music reveals a new aspect of matter not only by its action on the hearer, but also because the musician uses instruments more complicated than those of plastic artists.

The manufacture of these instruments is also a plastic activity which imparts definite forms to certain materials and enables us to study the effect of these forms on the sounds which we wish to produce. Between the simplicity of a painter's or a sculptor's tools and the great complexity of a piano, an organ or a violin, there is such an enormous difference that a long succession of experiments in the control of matter by human will was necessary to invent and improve these instruments. This again manifested some unexpected qualities in the various materials used: it was not easy to find out what kind of wood or strings should be used for a perfect violin or piano. The combination and harmony of many instruments in a modern orchestra corresponds to the enormous variety of sounds required by this exceedingly complex form of artistic creation.

All this human work for artistic musical purposes was at the same time a study of matter, because the fabrication of instruments added much to our knowledge of the materials which were used. This plastic activity of the manufacturers of instruments was supplementary to the efforts of the composers and performers who by a succession of sounds contrived to express immaterial realities, namely emotions and inspirations.

Here, as in metapsychics, matter appears as an instrument of the spirit and ceases to be a dominating power superior to man, like the matter dealt with by astronomers, physicists, chemists, geologists and biologists. The metapsychical experience, consisting in the materialisation of something invisible and impalpable, is repeated at a higher level in musical art. Music materializes a spiritual reality, as the succession and harmony of sounds has a peculiar relation to the infinite variety of emotions and inspirations, which in their ultimate stage attain mystic ecstasy.

A life superior to the life of the senses is revealed here through sensations. At a lower level sensation enslaves the sensuous man, but for the artist it is subordinated to the spirit and expresses realities of a nature far superior to matter, strangely enough through material means. It is a sublimation of that life of feeling which in the beginning was common to men and animals. The knowledge of matter won in this way reveals mysteries which could not be guessed by plastic artists or by scientists. Chopin teaches a musical mind more about the relation between soul and body, between matter and spirit, than is represented by the most marvelous discoveries of biology and even metapsychics.

But the meaning of symphonies and melodies remains ambiguous and may be interpreted in different ways, so that there is in music a yearning for an even more adequate expression of mental contents. Such a tendency finds satisfaction in a higher art, which expresses not only emotions, but also thoughts and actions. This is the art of poetry, which deals with matter still subtler than that of sounds, as it consists in words and rhythms which do not even need to be uttered aloud to reveal their beauty, provided that they are clearly imagined. Words are the matter of poetry. A word may be thought or imagined without being spoken. If it is pronounced, a musical impression is added, which is not, however, indispensable to the poetic purpose.

Words are matter of a kind which is invisible, impalpable, beyond the reach of the senses. Still they are matter, because they can be used by the poet as material for rhymes and verses. Poetical art produces materialisations of spiritual beauty like the plastic arts. Poetry gives a form to the material of words and sentences, the poet is a plastic artist in a certain sense like the sculptor, the

painter, the musician, but the matter which he uses may be controlled by the mind without affecting the senses. It is sufficient to think the words in a certain order: thereby we express what is intended. We may pronounce or write the verses which the poet has made, but this external manifestation is not essential, and a beautiful verse has already all its beauty before it has been uttered, written or printed.

In writing or printing his verses the poet shares with others his own spiritual experience, and in doing so gives a definite shape to the words of the language. Sometimes he creates new words, new modes of expression, new ways of using words, and then he creates matter that has not existed before, in a sense in which almost every artist is able to create the matter of his art. Thus the painter may create new colors which do not exist in nature, the musician new instruments and sounds never heard before. They utilize for their creations already existing matter. In poetry we witness the real creation of matter by the spirit in a sense still deeper than that in which we speak of the creation of sounds by the musician, or of new shades of color by the painter. The words are not always made of pre-existing matter, especially when they are used in a new meaning.

Even a sound consists of matter subtler than a star or an atom, but the word is subtler still and more subordinated to the spirit, for it needs no instrument to serve the poet's purpose. We see that as we rise from one stage to another in the hierarchy of arts, we notice an increasing predominance of spirit over matter. The sculptor or the architect depends more on matter than the painter. The painter depends more on the matter of his art than the musician, because he mostly imitates models, while the musician creates them. The poet achieves the greatest

freedom among artists, and reaches the utmost limits of creative art, for he gives life to what has never existed.

Finally there is an art superior to pure poetry, because it is the final synthesis of all preceding arts, the living representation of an aspect of human life. Its matter is dramatic poetry with every kind of decoration furnished by sculpture, architecture and painting, sometimes combined with music, song and dance. This is the theatrical art, which must be distinguished from the mere staging of a dramatic work. For a stage manager the poet's text is the matter to which very much must be added in order to create a successful performance. We speak therefore of the creation of Shakespeare's Hamlet by a gifted actor, for instance, Sir Henry Irving. Shakespeare has only imagined the words of the text. Sir Henry Irving added the intonation, the gestures, the movements, and this is the highest art.

The stage manager may see in a dramatic work more than the poet himself could see. Sometimes the poet does not fully appreciate his own inspiration: the stage is the final test for the work of the dramatic poet. A stage manager uses all the existing means to express human life, which as a spiritual reality, is essentially immaterial. In the theatre we have three points of view which mutually supplement one another. We have actors guided by a manager who should be a perfect actor himself, acting in the play and showing to his companions how each part should be played, as is done by the greatest Polish living actor, Juljusz Osterwa, in his company called *Reduta,* which, within the last three years (1925-1928), has given over fifteen hundred representations in more than one hundred Polish cities. Then there are the audience and the author. The author furnishes the matter, but it is

the manager who gives to that matter the shape which can satisfy the spectator.

Every spectator has opportunities for reflecting on the relation between the stage and life, between matter and spirit. The rehearsal of a play is a test of integral art with the utilisation of all arts in order to create the fittest sensations for the expression of an aspect of life. The aim of theatrical art is to lead us out of our own lives and to initiate us into the life of others as they live it on the stage. This stage life is the artistic condensation of human experience, derived from the actual living of those who resemble the characters presented. Whoever has not learned to enjoy a good theatrical performance knows real life only from his own narrow experience. It is art alone that gives to matter the new forms by means of which the accumulated life experience of many generations may be expressed.

Thus art penetrates the mystery of matter more completely than science, the various sciences and arts forming a scale of more and more intimate apprehension of its nature. The growing knowledge of the true nature of matter leads us at the same time to a deeper understanding of the spirit, as we see that the more we advance in our intimate knowledge of matter through the sciences and arts, the more it becomes evident that matter depends on spirit and exists in relation to spirit. The matter of metapsychics is subtler, less overwhelming, than the matter of astronomy. The work of the poet, the gesture and intonation of the actor, the staging of the stage manager, deal with matter still subtler, still more obedient to human will than the stone of the sculptor or the architect. This whole long growth of art teaches a lesson about matter that could not be supplied by science. It plainly shows matter as an instrument of the spirit, who may either materialize

his conceptions, or dematerialize existing things, until an unspoken word, a gesture merely imagined, take their rank as material objects because they act on the senses, or on sensual imagination, either positively or negatively.

3. *Industry.* The final mastery of the whole bulk of matter on earth is however achieved not by the fine arts, but by industry or technology which shapes and transforms material objects for purposes of human usefulness. Modern industry changes prodigiously the whole material world, creates new kinds of matter and teaches new uses of already known matter, changes our conception of space by enabling us to travel at very high speed, produces wealth to an unprecedented extent, increases our freedom of doing things undreamed of by our ancestors, enlarges perception by extending sight and hearing to distant objects, deserves, therefore, to be recognized as an independent method of controlling matter by the spirit and revealing, thus, to human understanding the true nature of matter. Industrial activity requires an amount of social organization not needed in science or art. We here have to deal not only with the lifeless matter, but with the bodies and movements of the workers, and industrial efficiency requires a thorough knowledge of this new aspect of matter.

The power of organization is a spiritual power, and if it is shown that it can considerably increase the output and improve the efficiency of the workers without increasing their numbers, we again have a case of materialization or giving a new shape not to passive matter as that of the artist, but to the cooperation of large numbers of living men, each controlling his own body. While the artist or the scientist act individually, in industry we have an eminently social activity directed by specially gifted leaders, for whom the industrial population is the immediate instrument enabling them to control matter.

4. *Asceticism.* Science, art and industry, however, are not the only ways of gaining experience of matter. They all deal with external matter, which is naturally independent of us. In science we seek to perceive and to understand what this matter is; in art, we seek to shape it and thus to transform it, in order to produce beauty; in industry, a similar transformation is undertaken for purposes of usefulness. In all these cases we deal through our body with something that is outside our body. But our body is also material and is the source of sensations much keener than those which come from without. Sexual enjoyment is a sensation, no less than headache, indigestion, heart palpitation, fatigue, exhaustion, hunger, thirst and even death. The control of all these sensations belongs neither to science, nor to art, nor to industry, but to a fourth great human activity, which is called asceticism.

The majority of men are the slaves of their sensations, they submit to every craving of the body without any attempt to control their senses. A long succession of experiences and efforts is needed in order to emancipate these slaves, to subordinate the body to the soul. But it is not the scientist nor the artist nor the engineer who succeeds in subjugating the beast in man. There is a fourth kind of expert, totally different from these, who struggles with his body in order to conquer it, namely the ascetic.

The first stage of asceticism is a negative discipline. It consists in eliminating from life every fleshly indulgence through fasting and mortification. The ascetic limits sleep and food, wears hair shirts, inflicts on himself pain of every kind, and generally deprives himself of everything pleasurable. He learns in these exercises to renounce many things which seem to most men indispensable. He

treats the matter of his body as an enemy, and this matter is no longer a color or a sound, not even a word—it is called temptation.

Temptation acts on the senses, is an object of sense perception or sensuous craving; therefore it is matter, as truly as stones and shapes, and colors, and sounds, and words. But if it is already difficult to identify sounds with the matter of stones and more difficult to realise that an unspoken word is also a kind of matter, it seems almost incredible that what we call temptation may be the same old adversary of the spirit, known otherwise as matter.

Yet we shall remain ignorant of matter so long as we have not taken this last step, and so long as we do not recognize in the ascetic, conquering the temptations of the flesh, as worthy a competitor for solving the mystery of matter as the artist, who gives shape to external matter, or as the scientist who investigates the material universe. Temptation consists in every sensual desire which confronts the will of the ascetic. It is not a visible or palpable shape, nor an invisible sound, nor an unpronounced word, but it is as real as any of these, though existing only in imagination, as do the words in our thought before they are uttered.

A temptation to which the ascetic does not yield is like an unspoken word, and still it is material as it acts on the senses and produces passionate desires. The temptation of the ascetic is matter even more subtle than the unspoken word of the poet, but has a tremendous material power which threatens the spirit with extinction. The aim of the ascetic is to control that matter by completely annihilating it. If he yields, he creates himself new matter that has not existed before, as every accepted temptation produces a long chain of other temptations which could have

been prevented had the first temptation been destroyed when it arose in the mind.

Above this negative discipline there is also a positive training which leads to the control of the body and to the elimination of temptations. While the negative discipline weakens the body in order to conquer it more easily, positive asceticism fortifies and beautifies the body through graceful movements, appropriate diet and hygienic care. This is the domain of gymnastics, including respiratory exercises, all kinds of sports and games, and also that perfect dance imagined by *Valery,* if we take it from the point of view of the dancer, not as a spectacle for onlookers.

Positive asceticism, if it builds up a body healthy, strong, beautiful, chaste, skilled and controlled by the will, penetrates more deeply into the mystery of matter than the negative discipline which conquers a body weak and well nigh exhausted. Positive training of the body subdues temptations quite as effectively as starvation and self-inflicted tortures.

In order, however, to understand fully the relation between mind and body, and the intimate nature of our own body, a positive experience of sexual life is necessary.[3] The difference of the sexes creates special temptations, which may more easily be avoided by running away from them than by bravely facing them and transforming them through a certain sublimation of feelings. Conjugal chastity is more difficult than monastic chastity and teaches us more about the nature of our bodies.

If the union of the two lovers, or husband and wife, occurs without any consciousness of sin, it is chaste de-

[3] The whole problem of sexual life and of the final aim of sexuality is treated more amply in the author's *World of Souls,* Chapter VII: *A New Theory of Sex* (pp. 196-220). There also the mystery of conception as a spiritual reality is explained, and a possible conciliation between the classic and the romantic view of love and marriage is attempted. See also the author's *Preexistence and Reincarnation,* pp. 45-55, 66-67, 118-121.

spite the greatest intimacy and enjoyment. Impurity is in our conscience, and the verdict of our conscience is not arbitrary, it does not depend on our desires.

If our conscience does not reproach us for an act which is usually condemned, then the particular act in question really belongs to a different kind, whatever its outward appearance. The sexual union is chaste if it is experienced as the mystic ecstasy of the lovers, very much above every impression of the senses, or any craving for sensual pleasure. Here the lovers themselves are the most trustworthy witnesses of their experience, if they are sincere, and not those outsiders who see only their outward appearance and cannot share the true reality of their common inspiration. Such a chaste and pure conjugal union is something totally different from yielding, after a struggle of the conscience, to impure temptations.

Voluntary procreation of strong, healthy and pure bodies is to be distinguished from the involuntary procreation of bodies with the hereditary taint of carnal passions. Carnal passion is the desire to enjoy sensually a body, while sometimes an act apparently the same, may really be a joint prayer of two souls for the noblest offspring, reinforced by complete renunciation of every selfish enjoyment and by the sacrifice of two lives for a third in conformity with the will of God. Voluntary procreation in a chaste conjugal union is the highest materialization, the art of shaping living bodies by attracting to them souls of a kind superior to the parents. It teaches us more about the essence of matter than any other kind of asceticism.

Sexual life, the closest intimacy of two persons of different sex, reveals certain aspects of the relations between body and mind and manifests either the superiority of will and mind over body, or the degradation of the soul.

This peculiar asceticism secures an emancipation from the yoke of senses such as cannot be achieved by the mere avoiding of opportunities of contact. The transformation and sublimation of impure desires into the spiritual rapture of a perfectly chaste procreative union, in which the real and sincere motive is the humble offering of a body to an incarnating spirit of the highest kind, abolishes the tragic consequences of the terrible prehistoric catastrophe known as the Fall.[4]

Thus asceticism repeats the lesson of science, industry and art, by showing us again the relativity of matter and the true controlling destiny of the spirit. It remains to decide whether there is a fifth way to study matter, which will supplement asceticism in a similar fashion to that in which art supplements science. Science is passive, art is active. Science observes, calculates, predicts. Art transforms matter, gives shape to that matter and even creates it. If asceticism acts on the body actively, transforms and perfects the body, how shall we name the corresponding domain of a passive perception of such matter as a Saint's body becomes through asceticism? It is cult, or worship according to an established ritual, in which material objects have an important place, as they are supposed to concentrate spiritual powers.

5. *Ritual.* Matter may become in certain exceptional circumstances a receptacle for spiritual force, if we accept the authority of some sages of the widest experience who have introduced a peculiar use of material things into the ritual of religious life. We find faith in relics among all peoples ever since remote ages. A relic is a material object supposed to be impregnated with spiritual force by a powerful spirit.

[4] The Fall or original sin is dealt with by the author in his *Preexistence and Reincarnation*, pp. 118-121.

There are relics not only religious, but national or personal. The pen of a great writer, the sword of a great conquerer, are worshipped like the bones of a Saint. In all such cases we imagine that matter, which has for long been subservient to a powerful spirit, retains something of the power of that spirit. Thus some people are fain to believe that a pen, which in the hand of a genius has served to write masterpieces, may help another inferior writer to rise above his own natural level.

This presupposes a concentration of purely spiritual forces in a material object. It may at first sight appear to involve a somewhat materialistic conception of a spiritual force, acting like an electric current. But if spirit acts on matter, and matter reacts on spirit, we may easily conceive that action corresponds to reaction in such a way that matter rendered thoroughly submissive to a powerful spirit becomes thereby able to transmit the power received to other spirits, just as a vanquished enemy may be incorporated into the army of the conqueror.

In the eternal warfare between matter and spirit, spiritual beings are sometimes dominated by their bodies; but, also, some material bodies may be deprived of their power of resistance and completely dominated by the spirit in a lasting manner. This faith in relics is almost universal and has wide applications. A relic is not only a single material object but may be an abode in which certain spiritual forces were once active. The walls of a church may be influenced by the prayers of generations in such a way that they become an active help to prayer; the pulpit from which famous sermons have been delivered may help a preacher. Also the walls of a gambling den may incite men to gambling just as the boudoir of a great courtesan may awaken or increase voluptuous desires.

Thus we constantly create relics by our attitude of

mind. A wedding ring is a relic supposed to be impregnated with the force of the solemn vow of husband and wife, and may help to conquer temptations against fidelity. In all these cases popular opinion supposes spiritual forces fixated in matter.

Matter here acquires a new aspect never suspected by science, industry, art or asceticism. It resisted the efforts of the scientist, the engineer, the artist, the ascetic. We are accustomed to distinguishing matter from spirit as an adverse force. The faith in relics which is instinctive in many even irreligious men, suggests the contrary possibility of matter becoming a positive auxiliary of spirit. The bones of a Saint are supposed to work the same miracles as the Saint himself would have worked.

At first sight this may appear as a superstition devoid of any foundation. But in the light of the whole succession of efforts in art and asceticism to conquer matter, the final utilisation of matter for spiritual aims is the outcome of all the preceding stages.

We have seen in art and asceticism the growing mastery of the spirit and the gradual dematerialization of matter, until it became such an interior reality as an unuttered word or an invisible temptation in thought or imagination, without outward manifestation. In asceticism the body grew more and more the instrument and expression of the soul. Why should all this age-long struggle not lead to a final victory? Already in metapsychics and asceticism the body seems to be partially annihilated, as for instance when it levitates in defiance of gravitation.

When material relics are charged with spiritual power the last stage of that process is reached. Matter depends naturally on the spirit, as it has been created for spiritual purposes: it was, according to the oldest tradition, the

creature that rebelled against its Creator and was punished by the humiliating yoke of the flesh.

Gradually the material world is being redeemed and made a relic of spiritual power. The use of matter in religious worship is the fifth way of initiation into the mystery of matter, besides science, art, industry and asceticism. Science and worship are passive; art, technology and asceticism, active. There is an analogy between the relation of art to science and asceticism to worship.

The belief in the possibility of concentrating spiritual powers in material objects finds its most thorough applications in the Catholic doctrine of sacraments. A sacrament is a visible sign of invisible grace. It is a kind of matter that is even more completely controlled by spirit than the body of the ascetic or the athlete, because the matter of sacraments is not subject to temptations and has a force that cannot be lost.

This sacramental force is transmitted through action different from that of the scientist, the engineer, the artist or the ascetic, and the function of matter in the sacraments is far superior to that in science, art, industry or asceticism. A sacrament is a relic suddenly created by the act of will of an expert, specially trained for this power by the exercises of priesthood.

The effect of sacramental consecration in the sacrament of Eucharist is called transsubstantiation. According to Catholic doctrine this change is not observable from without, it is concealed within the unchanged outward appearances. This is just what happens with every relic. A sword worn and used by a great general does not change to outward observation. The courage, prudence and wisdom of the owner are supposed to act on it mysteriously so as to produce courage, prudence and wisdom in others who will wear and use the same weapon.

Materialists will deny the possibility of such a supernatural force in relics and sacraments. But those who believe in that force and use it, attain a higher conception of matter as finally conquered by the spirit. If that use of matter were not already universally known, we should postulate its existence after our survey of the four other modes of apprehending the intimate reality of matter. If art supplements science, there must be something that is in a similar way supplemented by asceticism. This is generally called ritual, and consists chiefly in consciously creating and skilfully applying relics or sacraments for the moral improvement of man.

The conscious production of relics by rules of conduct in religious worship differs from active asceticism in much the same way as science differs from art. We have in the above survey put science before art, though art is much older than science,, because science has created the atomistic conception of matter, which is shown by the consideration of the four other ways to be thoroughly inadequate.

Out of the immense wealth of human experience in respect to matter, atomism takes into account only the oldest science, namely astronomy. It is not a whirl of dust, nor the contemplation of the stars, nor even the spectacle of fascinating Bayaderes that will reveal to the thinker the deepest mystery of matter. We approach this goal through asceticism, and find its most sublime symbol in the Holy Grail or in the Host exposed to the adoration of the faithful as the body of Christ.

The body of Christ is a body in which there is no longer any rebellion against the omnipotence of the spirit, and which becomes therefore a source of spiritual power. Matter, created by the Fall of angels and men, which produced beasts and stones, reaches at last a nobler purpose

when it is transsubstantiated and its baleful influence is gone. The religious conception of matter penetrates reality more than scientific atomism, the efficiency of art and industry, and even the most complete self-control of the ascetic. The value of this conception is objective and can be appreciated also by those who do not themselves believe in relics or sacraments.

6. *National cult.* A peculiar extension of the religious conception of relics and sacraments is supplied by the Polish theory of national life,[5] according to which the consciousness of a national mission not only transforms all loving beings engaged in that mission but operates a sacramental transsubstantiation of the whole territory in which a group of spirits akin to one another realises the kingdom of Heaven on earth through the happy cooperation of genius and sanctity.

In such a country every material thing will become a holy relic radiating spiritual power, and matter will cease to be a source of temptations. But long before this goal is reached, every individual who transforms his own life through the awareness of his participation in a national mission, will at once observe a change of the meaning his own body has for him and a new attitude of mind towards the whole material world on which he acts through his body.

This experience being as yet new in the life of mankind and exceedingly rare, it is impossible to explain it to those who do not share it, and the mere mention of such a reality not accessible to most readers has to be excused by the common observation that in every science a new fact is mentioned first by a single observer, before others confirm it and it becomes universally known.

[5] This Polish theory of national life has been stated in English for the first time in the author's *Preexistence and Reincarnation*, London, 1928, pp. 108-117.

At present this change of attitude towards the material world through the influence of national consciousness and the fixed will to return many times to earthly life for the fulfillment of a national mission, is known to a small group of spirits in each true nation. National life is the last stage leading mankind to a final understanding and experience of matter after science, industry, art, asceticism, cult and ritual.

Each of the earlier activities described was related only to a certain amount of matter. It is only national life that implies the whole matter of a country. If national life embraced the whole of mankind, changing hundreds of races and populations into a few true nations, being like organs of a body, then the national life of these nations would act on the whole earth with everything that is material on earth, until all this matter would acquire the quality of a religious relic, or even until the whole matter of the earth, including the bodies of all men and animals on it, would undergo a transsubstantiation similar to that which is taught in the doctrine of the sacraments.

This gives us the final solution of the problem of matter and also allows us to define the fundamental relations between soul and body. The whole of the matter on earth has been created by the spirit for the use of man, in order to give stability, permanence and objectivity to his subjective designs.

It has been spoilt by sin and grows in its evil power whenever we yield to temptations. But genius and sanctity, through art, asceticism, cult and ritual are able to regenerate matter until it again serves the original purpose of the Creator in artistic, religious and national life.

It is vain to attempt a definition of matter in terms of something else, as, for instance, bodies in movement. Matter is one of the fundamental categories through which

the mind attains reality, and it is understood by immediate experience, not by definition.

This understanding is furthered by the classification of activities in which the spirit uses matter; all these functions of the spirit in relation to matter may be reduced to two fundamental powers of perception and materialization. Things are related to one another either actively or passively. We receive from without certain impressions and this is called perception. We act on matter from within, producing definite shapes, or changing its inward nature, and this is called materialization.

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REASON AND NATURE

THE relation of reason to its natural setting, of knowing to the physical world, furnishes philosophy with one of its most crucial and difficult problems. In regard to the main question involved, whether the understanding is wholly part of nature, the answers, despite endless minor differences, seem ultimately reducible to either yes or no. That is, in the end reason must be regarded from one or the other of two points of view. On the one hand, it may be considered as simply one of several natural capacities (or as a function of them) marking by its presence a tendency of the organism toward abstraction, comparison, and reflection. Or, on the other, it may be taken as something preeminent and unique, as a capacity qualitatively distinct from and authoritative over the special aptitudes, and as lending man his peculiar supremacy over nature. Reserving discussion of this latter view for the time being, let us begin with an examination of reason as naturalistically conceived.

The developments of modern science seem to have shown, at least to the empiricist's satisfaction, that man's mind no less than his body is wholly of animal extraction and a part of the world of nature. This being taken as so, the same great evolutionary processes, the same physical and chemical laws which govern the rest of the physical world are held exhaustively to explain

the constitution of man. Just as animal life is definable in terms of its physical and chemical constituents, hereditary tendencies, and environmental situations, so human activities likewise including knowledge are held to be interpretable as functions of these same conditions. Accordingly, from this point of view, what appears as the mind's free selectivity, its power of abstracting certain features from the stream of experience and of noting their likenesses and differences, is really nothing more than a mechanical response of the organism to its physical surroundings. In other words, the organism is said to abstract or select just those features of the total situation that impinge upon it with greatest strength and intensity; thus we inevitably react to the brightest colors and the loudest sounds or at any rate to whatever stirs most strongly our organic needs and impulses. What looks at first sight like a process of self-determination in thinking turns out to be only, broadly speaking, a mechanism of natural selection; and all man's so-called intellectually creative and constructive "action" proves on closer inspection to resolve into so many forms of "re-action." The tendency to irritability, to motor response, apparently constitutes, from this standpoint, the fundamental differentia of life in all its varying forms. At the basis of life, we are told, stand the class of substances known as protoplasm, which are extremely instable compounds. Given these unsteady compounds of C, H, O, N, P, and S, with their propensity to variation, and by an inevitable process of interaction with the environment certain of them break down, whereas others better adapted to the surrounding physico-chemical conditions persist and win relative equilibrium and stability. This tendency of compounds to maintain their equilibrium as against their surroundings, their "inertia" or resistance to change, comes to be distinguished at the organic level as a definite pro-

pensity of things to "persist in their own being" (*conatus essendi*) or as a specific impulse to self-preservation. Accordingly, all the actions of living beings are to be interpreted on this view not merely as physical and chemical reactions but also specifically as "saving reactions," as mechanisms directed upon the preservation of protoplasm as protoplasm, of life as life.

But if this account is correct in its essentials, the reasoning and intellectual life of man are nothing but so many determinate resultants of physico-chemical laws. Even the most complicated activities of the highest developed nervous systems have to be accounted for in strictly mechanistic terms. Of course, owing to the incompleteness of scientific knowledge at the present time regarding the processes involved, wide differences in emphasis and detail are to be expected among empirical explanations. In certain quarters, for instance, chief stress is laid upon the concept of the reflex arc as the unit of functional activity, and all higher manifestations of organisms are reduced to the compounding of such arcs. Other hardy empiricists prefer to dilate especially upon recent discoveries connected with the ductless glands and the astonishing variations in psychic life apparently attributable to their over-development or atrophy. Through this means they find a way to explain man's reflective activities in terms of the chemistry of the body and its internal secretions. Still others incline to stress photo-chemical changes known as "tropisms" in lower forms of life. The turning of the sun-flower to the sun or the flight of the moth to the candle they take as expressive of the essential principle of behavior-reaction to which man in the highest reaches of his thought must conform. But since, despite various unbridged gaps and divergences in detail, these hypotheses agree in holding all activity to be explicable as a conditioned response to a particular stim-

ulus, they are obliged to construe rational knowledge also (as a form of activity) as the expression of a positive or negative reaction on the part of an organism toward some definite physical obstacle or incitement in the environment.

From this point of view, man's most magnificent inferences, his dreams and theories, his scientific laws, his engineering feats, and modification of species have all to be interpreted as so many complex adumbrations of fundamental organic needs, as ingenious adaptations or working devices accidentally evolved in the struggle for existence. In the last analysis, we are led in fact to believe that the sole abiding worth of man's religious, scientific, and aesthetic constructions is to be found in their contributory function to life as life. And although this extreme conclusion is sometimes disavowed by evolutionary naturalists, the implications of their doctrine are such that nothing else can well be meant. Theories, no less than claws, wings, and tails, are finally evaluated in terms of the generic problems of survival which they help to solve. The reason or intellect, no less than the leg or arm, is taken as in structural principle only another weapon of refined musculature wherewith to wrest subsistence from a recalcitrant world. Upon these assumptions, knowledge is essentially preservative rather than creative, a defence or acquisitive reaction to a particular stimulus, rather than an originative enterprise for reshaping the materials of experience to some pattern of the ideal.

But if man and his capacities are wholly part of nature, and if nature is an aggregate of sensuous particulars, then clearly man can claim no genuine knowledge other than of particulars or that rises above them. Hence when reflection seems to soar above the world of special *de facto* considerations and to concern itself with cosmic problems as if it were a universal spectator, let us not be deceived, says the

empiricist, but let us remember then man is simply an animal like other animals, a chemical compound like other compounds (for there is no element entering into his composition which is not common to the inorganic world,) and that as such he is constitutionally oriented first and last upon his organic needs and the maintenance of the stability of his physical system. Accordingly, naturalism denies the pretensions of reason to envisage genuinely formal and universal, as opposed to material and particular, objects. Concepts or generic notions are accounted as nothing more than "generalizations"; while theoretical grounds and reasons are denied causal efficacy, being construed as idle, compensatory "rationalizations" after the event. The real forces guiding thought and action are held to be those of our physico-chemical constitution; yet because we are unaware of this control we often mistakenly attempt to justify our irresistible motor tendencies by conjuring up *post facto* speculative grounds and ideal explanations of them. The plain fact is, according to naturalism, not only that man is unable to envisage true universals, but that (even if he could) he would be incapable of responding to such ideas as a stimulus. Accordingly, he must reconcile himself to being merely the product of certain specific mechanical conditions; and must comprehend his moments of apparent initiative, spontaneity, and origination as only the effects of deeper-lying causes in the order of nature.

In the extreme form in which it is here represented, naturalism is open to obvious criticisms. The chief objection to be offered is, of course, that these ends (life for life's sake and the maintenance of the equilibrium of systems, *etc.*) upon which nature is said to be directed are themselves metaphysical and teleological constructions of man's reasoning *about* his supposed animalism, chemical constitution, and so on. Here is more than a suggestion of

paradox. For naturalism, despite its disclaimers of all but the strictest empirical and scientific accounts of nature, finds that its very position, if systematically adhered to, constitutes a teleological-metaphysical theory about the world which goes far beyond the warrant of direct experience, yet which seemingly must be granted if the results of the sciences are to be construed as either trustworthy or significant.

Furthermore, question may well be raised regarding the view of thought as an instrument, on the ground that, even though in some sense it may be one, thought is so different in kind from the sense organs, from legs and arms, and other particulars of biological equipment as to be hardly comparable with them. The chief mark of most biological tools is the fact that they are bound up with the structure and locus of the organism. The leg, for instance, is attached to the body, and where the body cannot go, the leg cannot go; similarly with the others. Furthermore, they apparently require some direct contact with the environment for any experience or knowledge of it. Thus even in the case of an organ of distance reception like sight, the environment has to come to it. That is, specific vibrations must be given off by the object visioned, must be transmitted by the ether, strike the retina, be carried to the optic nerve, and so on; in short, the experience seems to presuppose something like adjacent or contact action through space between the stimulus-object and the responding organ. In thinking, on the other hand, such direct conjunction or overlapping appears unnecessary; and, as a result, questions of motion from place to place and bodily behavior become far less important. A thinker may presumably sit quietly with closed eyes and conceive events in Betelgeux or what will happen in the year 2,000 A. D., or review the age of reptiles in prehistoric evolution. In such cases

thought claims to grasp objects in the past, present, or future environment with which the organism has not, and in all probability will never have, any direct sensible contact. In thus prospectively delineating a state of non-experienced experience and retrospectively describing what the world looked like when there was nobody to look at it, mind seemingly assumes its power to transcend the narrow boundaries of direct acquaintance which circumscribe the organism, and to make use of an organon of knowledge distinct from any form of bodily behavior. Of course, it may be flatly objected by some that thought is mistaken in its presuppositions; and that, being only as it were a feeble chemical glow of an animal sensorium on a minor planet, it cannot possibly reliably envisage such remote and transcendent objects. Yet if this be so, and if thought cannot be trusted to mean what it claims to mean as regards its basic postulates, the entire structure of knowledge seems threatened, with the result that not merely thought alone but all its discoveries (including planets, sensoriums, organisms and environments) appear thrown open to question.

Before taking up these difficulties more fully, however, let us note the existence of certain modified forms of naturalism of wider scope than the foregoing. Many of these broader interpretations recognize the genuineness of reason as a distinctive aptitude and, though still denying its supremacy, admit its parity with the other functions of mind.

From the standpoint of a more liberal naturalism, one of the most striking marks of reasoning as compared with the other capacities of mind is its range of comprehensiveness coupled with an apparent economy of effort. Thus the objects of reason do not require full pictorial representation in consciousness as do, for instance, objects of memory, imagination, or perception. These latter are only held to

be trustworthy when they can be presented as particular existences before the mind in considerable detail. Yet if we credit current psychology, our range of attention is limited to the apprehension of some five or six discrete objects simultaneously, so that we are plainly handicapped in process like memory, imagination, or perception, by the time and effort required to marshal a small number of presentations on and off the stage of consciousness. Fortunately, however, there is another aptitude that is largely free from this requirement. Reason or understanding has the power to arrive at conclusions regarding its objects without the laborious, time-wasting necessity of grasping them as particular existences and holding them individually before attention. This is because it is able to lay hold of the form or abstract schema of objects as distinct from their particular content. Thus reason with its capacity for representation through the relational structure of ideas rather than their matter seems to offer quite incredible resources for the enlargement of knowledge; vastly wider possibilities of synthesis, in fact, than could be won presumably by memory in history or imagination in art.[1] This greater scope is also, as was said, correlated with greater saving of time and energy. Were it not, indeed, possible for reasoning to dispense with most of the details of presentation in consciousness, we should sit and perish while seeking to arrive at a small number of conclusions.

Another mark of reason duly recorded by a broader naturalism is the propensity to organize data into systems and to disclose interrelationships among objects hitherto apparently disconnected. Indeed, some have even gone so far as to define reasoning as just this tendency to interconnection persistently applied. In conformity with this in-

[1] Of course, this does not deny that rational activity may include and make use of memory and imagination (in a way, in fact, in which they can never make use of it) but only stresses that it can never be identical with them.

clination, understanding never apparently accepts a "fact" off-hand at its face value or takes an isolated judgment as more than provisional; but requires that each shall be confirmed by linkage with other facts and judgments which mutually sustain and support it. My belief, for instance, that "This azalea is red" appeals for tacit corroboration to the body of my sensory judgments, to their power of correcting and supplementing each other, to the verdict of competent witnesses, and so forth. Moreover, each group of consilient judgments seems to lead on to other groups. In the end, although reasoning presumably never realizes the ideal which is that of a single, all-inclusive system with no grounds outside itself, it is customary to assume that, other things being equal, the more comprehensive a coherent body of judgments is and the richer in interconnections, the more reliable it is likely to be. No other capacity of mind appears comparable to reasoning in respect of this power of organizing the data of experience, on the one hand, and, on the other, of performing the equally valuable negative function of exposing contradictions.

But even such broad, eclectic descriptions of reason as the foregoing are open to the charges preferred against naturalism, as it seems to me, in so far as they deny the priority of reason to experience. It is not enough to distinguish reasoning from the other capacities merely in degree, or to note its superior aptitude for synoptic and symbolic representation as compared with them. Its supremacy over the rest of mind must also be recognized, together with its power of illuminating the objective order of things. Only by supplanting the foregoing views with a second view of reason, regarded as logically prior to experience and in so far possessed of a supranatural character, can the contradictions of naturalism be avoided.

The paradox of naturalism rests, if I am not mistaken,

in its assumption that the mind and its constructions can be wholly included as a finite part within the sphere of nature. For any attempt to explain the mind and reason as the product of a naturalistic process must tacitly allow the self-refuting assumption that the process described is itself the product of mind. In other words, despite itself the intellect comes to be admitted as both the source and the product of nature. Nor is the contradiction to be avoided by taking refuge in the distinction between the *facts* of nature and the *theory* about them, and claiming that only the latter is the mind's creation: for this very distinction is itself a construction of mind. Had not the mind been adequate to comprehend evolution as a theory, we should have no reason whatever to believe in evolution as a process. Moreover, once naturalism can be brought to see that the nature of nature (*i.e.* its laws and operations) is disclosed only to intelligence, and that our contact with facts is always in a context of theory (admittedly of the mind's creation) it surely cannot deny, in the absence of all negative instances, that nature as the object of thought is inseparable from the activity and construction of thought which reveals it.

Apparently the only way in which thought can escape the contradiction of conceiving the intellect as both the source and product of evolution is through the frank recognition of the self-transcendent competency of reasoning to raise itself above the natural order and to be the spectator of its own development in experience. But if, on the contrary, we accept the naturalistic account of mind and endeavor to limit our thinking rigidly to these assumptions, the riddle becomes insoluble how so fragile, uncertain, and accidental a phenomenon as intelligence should be qualified to pronounce a verdict or draw credible conclusions regard-

ing the nature of things in general. What warrant can we have of the reliability of the human mind as a criterion, when by its own admission it is so deficient and circumscribed in power and extent? What guarantee have we of the strictness of its leadings or the veracity of its conclusions?

Broadly speaking, the tenets of naturalism as apparent in present philosophy may be summarized somewhat as follows: First, all knowledge is derived from experience by methods ultimately empirical, and nothing can be said to exist save what is definable in terms of experience. Second, the favorable maintenance of existence and the stability of systems appears to be a fundamental tendency of natural processes. Third, the basic order of nature is temporal and causal and there is no separate order discoverable of logical dependence. Fourth, intellect or reasoning is only a *proper part* of nature, and as such is always less than and included within the *whole* of it. Our contention, on the contrary, is that naturalism, wherever it adheres strictly to the implications of these propositions, involves itself in a self-refuting position whereby knowledge itself becomes impossible. The only plausible escape from this predicament, as we see it, is through acceptance of the supranatural and transcendent power of reasoning, recognition of its capacity as an infinite part to encompass the whole, by which means alone the paradox of intellect (its inclusion within the object revealed by it) seems resolvable.

For if, as naturalism maintains, all knowledge is limited to experience man can hardly claim to arrive at any accurate estimates of universal processes of the world order, since in the nature of the case his organism can never wholly traverse or sensibly examine all parts of the environing totality, or for that matter even a single aspect

of it.[2] Yet if this be so, clearly man can have no assurance of the trustworthiness of the second proposition of naturalism, that survival value is the ultimate scale by which everything is tested. Such an assertion not only transcends the scope of empirical verification, but also expresses a metaphysical insight into the nature of the universe, which is precisely the sort of knowledge which naturalism disclaims as impossible. In the third place, when naturalism denies the reality of the logical order and reduces all processes to those of temporal succession it conflicts with science, and even contradicts its own conclusions in so far as derived from science. For wherever science establishes an hypothesis regarding nature, it does so by means of a reflective analysis working in reverse order from that of the temporal genesis assumed to hold in the natural process itself. Furthermore, the very formulation of the law or theory seems to imply that it is revealed to a logical spectator or disembodied intelligence which is able in a single *coup d'oeil* to survey the sequence of events in time and space. Acceptance of the evolutionary hypothesis plainly presup-

[2] Thus the propositions 'All water is H_2O' or 'Ammonia is N H_3' make assertion that go far beyond the empirical evidence of the cases examined, since only a very limited number of samples have actually been analysed. Clearly *mere experience* is not entitled to authorize a pronouncement here as to the nature of the *non-experienced* cases.

To this the empirisist may reply that the proposition only means that 'So far as experience has gone such has been the case, and therefore man has an empirically justified tendency to expect that future experiences will resemble past experiences.'

But that mere experience entitles us to make this kind of generalization involving past and future is precisely what the rationalist questions. The past and future for the radical empiricist, he maintains, must both be constructions from the immediately present 'given' of the organism; and as such they never fall within the limits of actual experience at any given time. What we call the past, for instance, is really the work of memory, which constantly selects and arranges sensory material in reverse order, daubing it with the light and shade of retrospective emphasis, and in general creating an extraordinary fiction of *experience as it was never experienced*. Even more obviously the futures which figure in our predictions are fictions respecting non-existent experiences, since we cannot by any twist of interpretation claim actually to have lived through future futures.

poses the power of mind to raise itself above the natural order. That is, unless we suppose the competence of intellect to outflank and encompass the natural process (unless we assume the part as equal to the whole,) it appears impossible to credit as trustworthy this same intellect's account of the world-process including its claim to be itself a product of it. The logical status of intellect must, if genuine knowledge is possible, be assumed to be independent of the conditions of the emergence of mind in the space-time order: since, on the one hand, the primacy and priority of reason must be granted in a logical sense before, on the other, the evolutionary account of its late appearance in phenomenal history becomes credible. Here, as elsewhere in the system of knowledge, the relation of causal sequence may run directly counter to the order of logical dependence.

But naturalism, in its zeal to construe man as wholly part of nature, apparently overlooks both the inconsistencies in its own premises and the presuppositions of scientific method. That the transcendence of mind over nature is tacitly granted by science in its procedure seems to us something that can hardly be denied, considering the logic of its assumptions. For only by presuming the adequacy of intellect to embrace the phenomenal course of events, is science able to place confidence in its own results. Were this power of transcendence denied, the belief in uniformity, in the past and future being really as they are thought to be—for that matter, the whole of inferred history and scientific hypothesis—would be undermined. Moreover, scientists themselves are today calling attention to the limitations involved in the strictly empirical or observational standpoint. Recent discoveries have emphasized the enormous biological handicaps that man is subject to in laboratory experimentation. Owing to the fact that the scientific

observer is always planted in a human body and upon a larger planetary body, from both of which he is powerless to detach himself, universality and objectivity can hardly be claimed for a particular set of observations made from a particular standpoint. For where the locus and activity of the observer are themselves part of what is observed (as is usually the case,) absoluteness can hardly attach to the individual results, which are bound to be colored by naturalistic peculiarities. Introspective methods in psychology, for instance, and experiments like those of Michelson and Morley in physics would seem to have shown by their failures the impossibility of determining the behavior or movement of a system by observations within the system.

Yet at the same time that science today recognizes that the observer cannot jump out of his skin in a natural sense, it nevertheless allows that he can stand outside his private viewpoint intellectually. That is, although we remain imprisoned within the confines of our sensori-motor system and chained to its locus as regards our immediate perceptions, we are still able by means of theoretical reckoning and calculation to discount these impedimenta through correlating the standpoints of different observers with one another according to definite rules, so that the laws of nature or ideal relationships disclosed may be freed from dependence upon the accidental features of individuality and hold not only for one but for a plurality of systems. This is only another way of saying, as I understand it, that science admits the competence of reason (although a part) to step outside itself and to assess the whole in which it is contained.

For how, unless the self-transcendence of reason be allowed, can a circle be avoided in the technique of measurement where so often the act and instrument of measure-

ment are part of the system to be measured? How, indeed, when in physics calculations are made with physical instruments which are consequently themselves subject to physical laws; when the products of research in the historical sciences are themselves a part of history? Mr. P. W. Bridgman has recently called attention to some of the logical difficulties of this sort within physics in a passage which it may be well to quote.

"In order to ascribe any simple significance to postulates about the velocity of light," he says, "it would seem we must have an instrument for measuring this velocity, and therefore for measuring time, which does not itself involve the properties of light. To do this we might seek to specify the measurement of time in purely mechanical terms, as for instance in terms of the vibration of a tuning fork, or the rotation of a flywheel. But here again we encounter great difficulties, because we recognize that the dimensions of our mechanical clock may change when it is set in motion, and that the mass of its parts may also change. We want to use the clock as a physical instrument in determining the laws of mechanics, which of course are not determined until we can measure *time*, and we find that the laws of mechanics enter into the operation of the clock. ... In practice, the difficulties of such a logical treatment are so great that the matter has been entirely glossed over."[3]

Escape from epistemological difficulties of this sort seems impossible, in our opinion, without frank recognition of the right of logical postulation and of the competence of the intellect to stand outside the different systems and to combine and cancel out, as it were, their individual peculiarities.

On any other assumption, the possibility of obtaining genuinely universal propositions would be very hard to explain; yet science seems to assert just such universal propositions. For instance any one who affirms that "all bodies

[3] Bridgman, P. W., *The Logic of Modern Physics*, pp. 70-71.

gravitate" is himself possessed of a body, and as such claims to come under the law he enunciates. Now from the standpoint of naturalism, it follows that if the formulator of a law falls within its scope, the law is open to suspicion. For since naturalism denies the possibility of transcendence, it can hardly do otherwise, when confronted with a clear case of the inclusion of the *part* which does the measuring within the *whole* which is measured, than question the authenticity of the results. Nor is the difficulty to be avoided by saying that Newton or the observer did not mean to include his own body under the principle of gravitation, since in that case he did not say what he meant; if by "all bodies" he did not mean "all bodies" but made an important exception of his own, then the vaunted universality of the law is unfounded. Similar contradictions are discoverable in the principle of the conservation of energy, so long as it is interpreted on naturalistic assumptions. That is, either the formulation of the principle must aim to cover even the particular amount of energy required in the enunciation of the law (in which case, we have a paradox) or else, if not intended to be included, the pretensions to universality are false. And the same thing holds of evolution. Man, when he formulates the proposition that "all life evolves" plainly claims that he himself falls within the process; (and what is this but to declare his body, mind, and whole scale of measurement part of the object measured?) Yet if this be so, what credence can be placed in the theory itself, in the feeble attempt of the creature to embrace the creative process, of the effect to reabsorb the cause? Acceptance of the naturalistic account of evolution with its emphasis upon the tardy, diminuitive, and casual appearance of mind in the cosmic sequence seems here in conflict with the assumed priority of reason as a presupposition of

scientific knowledge. Our conclusion is that so long as mind and reason continue to be used as the master-key to unlock the riddles of nature, unequivocal recognition should be given to the logical supremacy of the instrument over the object and its adequacy to compass the task.

A simple illustration may serve perhaps to make clearer at this point the relation of the biological and the logical orders and what is meant by the priority of reason over nature. Everyone agrees, we may assume, that the Mendelian law of independent assortment could not possibly hold in its present form unless hybrids were hybrids, unless the mathematical ratio of 3:1 meant precisely what it does in our number system, or unless the units contributed by the parents to the germ cells of their offspring followed certain atomistic principles of external relation;[4]—in short, unless certain laws regarding identity, number, and mechanism were valid. On the other hand, it seems no less evident that the principles of identity, number, and mechanism do not, as we understand them, invoke or appeal to biological laws to establish their reliability. While an organism with a brain that functions in definite ways is undeniably required for *thinking* these mathematical laws, the integrity of the laws themselves is in no whit affected or validated by the accompanying biological and psychological phenomena. Of course, wherever knowledge takes place a variety of more or less constant accompaniments is to be expected: spatio-temporal factors, bodies, oxygen, heat, movement, consciousness. Yet such factual concomitants are hardly to be mistaken for logical grounds or sufficient conditions of the process. For the latter, we must turn to the very different way in which mathematical and logical relationships underlie the biological world; here it is not

[4] Morgan, T. H. *et al.*, *Mechanism of Mendelian Heredity*, p. 1: "The units contributed by each parent separate in the germ cells of the offspring without having had any influence on each other."

merely that without the former the latter could not *exist,* but that without logical criteria to substantiate them biological laws could claim no credible meaning or validity whatsoever.

When inquiry is made as to the outstanding features of reflection viewed as a transcendent process, the answer is often that it places the significance of everything in its referable and inferable character. By this is meant that nothing is considered purely in itself or on its own account, but that everything is taken as the sign of something else. In other words, reasoning does not assume its object to be a bare datum, but on the contrary takes what is given as the representation of something not given, which serves it as evidence or support. The deliverances of sense, imagination, and feeling, on the other hand, take their stand primarily upon immediate experience, not sharing the assumption of reflection that data derive their significance from a source outside them. In the language of empirical procedure, "everything is precisely what it is given as, and is not to be explained away in terms of something else." So long as we stand within the actual sensuous, emotional, or imaginative experience, it raises no doubts as to the object, treating it as so much given fact devoid of extraneous implications. Now although in most of the enterprises of knowledge, both rational and empirical factors are so interwoven as to be scarcely separable, it is nevertheless possible to contrast the two in a broad way by equating them with the methods of induction and deduction. In the one case, description, in the other, explanation, becomes the ideal of knowledge.

Wherever we aim primarily at acquaintance with particulars, and are satisfied to learn about "some" without knowing about "all" members of a class, the empirical way

of looking at things is of the greatest value. Under these circumstances, observation, enumeration of instances, experiment, and practice play an important part. When, however, our intention is rather to obtain universal insights into the nature of orders or groups as wholes, the standpoint of concrete experience becomes insufficient, and has to be supplemented by some super-experiential means. Considering, as we must, the inexhaustibility of our world in almost any of its aspects, the countless number of beings of any kind one cares to mention (amoebae, crystals, stars, men) and the impossibility of our encompassing more than a trifling fraction of any given class, it becomes clear that, if we are to claim the right to frame universal laws and to deal with infinite kinds, we must employ some method whereby a limited amount of direct acquaintance can be made to suffice for a more or less adequate theoretical knowledge of the whole. In other words, sooner or later knowledge is forced to fall back upon deduction, upon the symbolic representation of whole by part, and the methods of rationalism.

An illustration from Leibniz may help to enforce the contrast here between rational deduction, with its capacity to master an infinite subject matter, and the impotence of empirical procedure to cope with other than a strictly limited material. When, for instance, we consider the series of squares of the rational whole numbers (1, 4, 9, 16, *etc.*) we may discover by direct examination that the difference between each square and its predecessor is an odd number, and that these differences when arranged serially appear to form the progressive series of the odd numbers. On the basis of this knowledge we are led to expect that, if we take a given member from the series of squares (*e.g.*9) and add to it the corresponding number from the series of

odd numbers (*e.g.*7), the result will be the next higher square in the series of squares (*i.e.*16). This expectation, however, is based upon merely empirical and psychological considerations; so that, no matter in how many instances we find it holds good, it still remains possible that at some further point in the number series the correlation will be interrupted. Only by adopting a rational deductive approach, instead of an empirical inductive one, is it possible to obtain evidence of a universal and necessary connection between the series of squares and that of the odd numbers. Such an approach discards particular numbers with their peculiar properties, and instead takes number in general; thus n is conceived as any rational integer. By means of n, the difference between any square and its predecessor may be expressed algebraically as $(n+1)^2-n^2$, which difference is $2n+1$ or the value of an odd number. Thus the universal, non-empirical formula $(n+1)^2-n^2=2n+1$ shows conclusively that the difference between the squares of any two successive integers must in every case be an odd number; and that a fixed connection subsists between the progression of squares and that of the odd numbers, so that (by means of the formula) the position of any given odd number can be definitely determined with reference to the series of squares. In brief, the totality of squares and odd numbers are shown to be linked together as parts of one system, each side of which can be known through the other. So long as numbers continued to be treated empirically and individually, no such discovery was possible, and they remained in a merely external, unexplained correspondence. Once an equation expressive of their essential relation was deduced, however, empirical tests involving the multiplication of instances were rendered superfluous. By the analysis of such examples, the incapacity of empirical methods to comprehend a universal object seems rendered plain, to-

gether with the striking capacity of deductive procedure to compass an infinite subject matter.

But while examples of this sort may be found with comparative ease in mathematics, the difficulty becomes much greater when we seek to find some method applicable to physical nature that will be adequate to compass its endless variety. Faced with its inexhaustible fecundity and the countless succession of individuals that embody every type (bacteria, dew-drops, molecules), the range of possible experience and empirical knowledge shrinks to a mere pin-point.

Yet confronted with this appalling subject matter so largely inaccessible to control by experimental methods, we seem almost driven, whether we will or no, to take refuge in some deductive theory. And here the analogy closest to hand, and the one which appears most acceptable to reason, is that of the game of chance. We face the world, as has well been said, as if it were an unending lottery from which we were obliged to draw handfuls of tickets in succession. By carefully noting the character, combinations, and frequency of our drawings, we seem to hope furthermore to be able to deduce the contents of the box as a whole. That is to say, we retain a certain confidence in the assumption that nature can best be treated as the permutations and combinations of a limited number of elements. It is to be noted that in approaching nature in this way as a combination of relatively qualityless entities in calculable relations, and in attempting to correlate the empirical record of things with theoretical predictions as to their probability, we are at least still relying on the logical structure of phenomena.

For our assurance as to the reliability of this technique is rooted in our knowledge that it is based upon the general laws of logic and their quantitative elaboration. At bot-

tom, "the theory of probability is simply," as C. S. Peirce has said, "the science of logic quantitatively treated."[5] Even when we hazard predictions on the basis of statistical data and proportional frequencies gleaned from experience, our mathematical expectations are ultimately derived by an inverse process from an *a priori* calculus. In forecasting the future, for instance, we are guided by the assumption that the situation is reducible to a limited number of equally likely, independent alternatives. And our belief in the method is strengthened not only by the fact that it appears impossible to conceive a world that should not conform to the laws of permutations and combinations, but in addition by the many results in the physical sciences which apparently exemplify the quantitative and atomic view of nature.

Thus, if these brief suggestions are not mistaken, the basis of the transition from *some* to *all* in our knowledge of nature is not an ungrounded leap of faith but the application to phenomena of the formal principles of combinations and probability. And once considerations of probability are recognized as fundamental, the process of "sampling" or what might be called "logical representation" of whole by part becomes also of great importance. When, for instance, we estimate the prospects of an unopened mine by picking up specimens of rock and ore scattered about in the vicinity, or when we sample a carload of wheat by examining handfuls from different parts of the shipment, we rely upon the principle that smaller groups, if chosen impartially and at random from a larger group, tend to have the character of the larger group. Having discovered from the probability curve that the combinations or chances which give us the mean character of a collection are more numerous than those which represent its extremes, we naturally conclude that those examples which appear most

[5] Peirce, C. S., *Chance, Love, and Logic*, p. 64.

frequently in experience are more likely to depict the dominant character of the whole than any others.

The same idea of the representation of the whole by the part can be seen to guide our predictions of the future. The oftener things have happened together in our experience, the more probable it appears to us that they will be found together generally in nature. Here, and elsewhere, in trusting the results of our partial, random experience to provide us with an approximately correct description of the qualities and proportions that subsist in the larger, unexperienced world of nature, we are depending upon the formal rule that the members of kinds that are actually more numerous in a large, varied collection have a greater number of ways of combining, and hence more chances of occurring than those members of kinds that are less numerous in the whole collection. For although the method of sampling is never beyond the risk of error, its mistakes often cancel out, and the process tends to correct and better itself. Broadly speaking, the larger the number and range of samples selected, the more the margin of error declines; while the greater the proportion of a given type of correlation, the greater the chances that a genuine law of nature has been discovered.

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INTRODUCTORY TO A MODERN PHILOSOPHY OF RELIGION

I.

THE term "Philosophy of Religion" is becoming more and more frequently met with. The use seems to go no further back than the classical German Idealism of the early nineteenth century, in which it had analogues in the phrases "Philosophy of History" and "Philosophy of Nature." Though used since by no inconsiderable number of German writers, it is only within more recent years that it has been widely adopted elsewhere. The earliest English book with this title appears to be the *Philosophy of Religion* of J. D. Morell, published in 1849, after which there was no other important work so described until John Caird's "Introduction to the Philosophy of Religion" which appeared in 1880.

The Philosophy of Religion is the successor of what in earlier times was called, sometimes Rational Theology but more often Natural Theology. The term "Natural Theology" is still retained in Roman Catholic circles today, as may be observed in Cardinal Mercier's *Manual of Scholastic Philosophy,* volume two, and in Dr. Boedder's *Natural Theology.* It was current amongst the English deists and their opponents in the eighteenth century. Dr. C. C. J. Webb has discussed at some length the various implications of the term "Natural Theology" in the first

part of his *Studies in the History of Natural Theology,* 1915. Natural Theology was contrasted with Revealed Theology at least in this, that it was regarded as knowledge attained by human reason unaided by divine revelation as usually conceived. It depended in part upon the examination of external nature, as might be seen for example in the famous Bridgewater treatises which endeavoured to describe intelligent (and so divine) purpose in the structure of particular parts of the physical world. It also depended in part on a rationalistic discussion of ideas, such as was developed for example from Descartes' treatment of the idea of God and from the first book of Spinoza's "Ethics." To some extent it was further related in part with the supposed natural feelings of mankind. Eventually, however, it tended to become a somewhat formal and abstract consideration of arguments for the existence of God and the validity of the ideas of human freedom and immortality.

The deists of the eighteenth century, mostly champions of Natural Theology, considered this to be concerned with the establishment or justification by means of reason of what they called "Natural Religion." Natural religion they regarded as a universal feature of human experience, shared by adherents of all religions. Anything that any historical religion contained in addition to these fundamental common elements was considered as unimportant and dispensable, or worse as a contamination of the simple pure religion of natural man. In this, as in their other views concerning man, the eighteenth century manifests its unhistorical and unevolutionary standpoint.

With the increasing use of the term "Philosophy of Religion" it is not often that we now meet with new books bearing the title of "Christian Apologetics." The impression has become widespread among theologians that some-

thing more thoroughgoing is required than is represented in the traditional literature of apologetics. They see the importance, the inevitable necessity, of philosophical consideration of the ideas with which they are concerned. But much of the older type of apologetics still remains masquerading as philosophy of religion to the discredit of the latter. The attitude of apologetics has been determined principally by the desire to defend and to promote a particular form of religion and particular theological doctrines. Apologetics has set out with certain formularies, such as the Christian creeds, and has endeavoured to defend them from attack and to find grounds for their acceptance. The attitude of the philosophy of religion is quite different. It does not set out determined by the wish to establish any particular results; it does not set out from any particular beliefs; and its methods are different from those of apologetics. Whether there is any place for the continuance of a modern form of apologetics we need not consider here: all that should be recognized is that the modern philosophy of religion implies a different attitude and has other motives. Inspired, as all philosophy, primarily by the desire for truth, it seeks this whether the conclusions it arrives at be favourable or unfavourable as concerns the persistence of any creed or religion or any part of them.

The modern rise of the Philosophy of Religion is in part due to a reaction from the dogmatism of these earlier apologetics. But it is to no inconsiderable degree also dependent on a reaction from the dogmatism of exponents of natural science whose views left no possibility for any interpretation of religion other than that of being a form of superstition. In opposition to this latter form of dogmatism the view has now gained wide acceptance that religion constitutes a distinct constituent of experience,

and demands explicit consideration in any complete philosophical survey of existence. The philosophy of religion is looked to as a legitimate and even necessary form of investigation, even while its form and methods are still uncertain. But notwithstanding this continued uncertainty, it does seem that there is gradually growing up a definite philosophy of religion of an undogmatic kind, different both from the earlier Natural Theology and from theological apologetics, in its starting points, its method of approach, and the general character of its working out.

Natural Theology suffered severely, some would say final defeat, at the hands of David Hume and Immanuel Kant. They both appeared to undermine the prevalent opinion that the existence of God and the validity of the ideas of freedom and of immortality were capable of rationalistic demonstration. The ontological argument from the idea of a perfect being to the affirmation of His existence as implicated in that idea; the cosmological argument from the world to a transcendent first cause; and the teleological argument from purpose in the world to an intelligent designer, were all shown to be open to serious objection on epistemological grounds and to be inadequate for the object for which they were used. It would be incorrect to maintain that there is nothing within these arguments which is of significance to the modern philosophy of religion. But even since the time of Kant they have continued to be used in much apologetic literature and in not a little that presents itself as philosophy of religion, in a manner which is subject to the criticisms which Hume and Kant expressed.

In different ways both Hume and Kant pointed beyond their own criticism to a more constructive attitude, though neither definitely worked out their suggestions with reference to the realm of religious experience as such.

While rejecting the usual theoretical arguments in support of religion, Hume in his *Natural History of Religion* turned to some consideration of religion itself and to what at that time appeared to him its history. But such investigation and the problems it has raised go far beyond what Hume himself discussed. On the other hand Kant's constructive effort was much more thoroughgoing and suggestive, but it kept in essence to the ethical and barely touches the distinctively religious. His work is of influence in the philosophy of religion nevertheless. For Kant's moral philosophy is dependent ultimately on the recognition of the distinctive data of the moral consciousness, even though he did not investigate it in the manner which a more realistic theory of ethics would demand in our own day. It is following out the fundamental principle implied in his moral philosophy if it is insisted that the philosophy of religion must rest primarily no more on the moral than the moral on the mathematical. While insisting on morality in its own right, Kant does not appear to have thought that religion may also with equal justification be regarded as existing in its own right. Yet it is only on such a recognition that any genuine philosophy of religion seems possible.

The essential implications of the contention made at the conclusion of the preceding paragraph have not been clearly and fully understood until our own times. After Kant and right up to recent years though the change from the title of Natural Theology to Philosophy of Religion has been widely adopted, there has been no adequate corresponding change in the methods of treatment. The changes that took place in the nineteenth century were more definitely to forms of speculative philosophy. It is worth while noting the character of some well-known works of the last few decades.

Dr. A. Caldecotte in his *Philosophy of Religion in England and America* (1901) says on p age 3 "Theism or the Philosophy of Religion" thus equating the Philosophy of Religion with Theism. Accordingly the literature he discusses in his wide survey is concerned almost entirely with the justification of the belief in God. Dr. J. Lindsay definitely restricts the scope of his treatise to Theism, entitling it *Recent Advances in the Theistic Philosophy of Religion* (1897). Almost all the literature he discusses is speculative and has little connection with or reference to the empirical religion. H. Lotze's *Outlines of the Philosophy of Religion* (Eng. trs. 1901) is mainly speculative and formal, though it breaks away from this somewhat in its treatment of ideas like those of salvation and of redemption. J. J. Gourd's *Philosophie de la Religion* (Paris 1911) is purely speculative, striving to interpret religion as the relation of the mind to an incoordonable factor in the theoretical, the moral, and the aesthetic.

Even works which do not entirely omit reference to actual religions have been too often developed on an accepted speculative basis. This is true in essence of H. Höffding's *Philosophy of Religion* (Eng. trs. 1906) in that his later considerable use of material concerning actual religions is related with a theory of knowledge (and virtually a metaphysical standpoint) elaborated in the first part of his book. The first four chapters of H. Rashdall's *Philosophy and Religion* (1909) are speculative along the lines of a Berkeleyan idealism, even though in his attention to Christianity in the last two chapters he gives some consideration of the empirical. *The Philosophy of Religion* (1912) of Dr. G. Galloway, and that of G. T. Ladd though they occupy themselves more definitely with the actual experiences and ideas of the

religions, are still predominantly speculative, and refer to these chiefly by way of illustration: they do not make an empirical examination of historical religions their starting point. Dr. Galloway's attitude may be seen from a statement in his preface: "A speculative theory of religion must be judged mainly by the fairness with which it interprets and the adequacy with which it explains the religious experience as a whole." But it may be asked whether, and if so how, such a speculative theory can be arrived at apart from taking "the religious experience as a whole" as the basis in relation with which it is developed?

Strangely enough we see a pointer to the newer form of the philosophy of religion in the works of Hegelian Idealists amongst whom the term itself most definitely began to be used. John Caird's *Introduction to the Philosophy of Religion* (1880) although itself purely and entirely speculative, nevertheless insisted on the need of "a detailed examination of the various positive religions." (p. 313). And in his Gifford Lectures: *The Fundamental Ideas of Christianity* he gave his attention at least to the main intellectual contents of this religion including the examination of such ideas as those of incarnation and atonement. It is, however, in the much neglected volumes iii and iv of Otto Pfleiderer's *Philosophy of Religion* (Eng. trs. 1888) that the importance of the recognition of the empirical factors is most clearly seen. These volumes are called the *Genetic Speculative Philosophy of Religion.* They include a critical and constructive consideration of the empirical facts of religion in the history of humanity. It is well to quote some of Pfleiderer's own words. In his general introduction he says: "Philosophy of Religion in the narrower sense, and strictly speaking . . . . is the systematic investigation and comprehension

of the totality of phenomena which in the life of man compose religion." Referring at the end of the second volume to the conditions of the study at the time he wrote, he contends that the historians of religion "are generally as destitute of any philosophical ideas as the philosophers are of historical reality." Then he goes on to indicate the aim of the volumes iii and iv of his treatise: "To work these two elements into each other so that each may afford a basis to and cast light upon the other; such is the aim of my genetic-speculative philosophy of religion." Pfleiderer made a noteworthy attempt, but his work is unsatisfactory on two counts. It is after all written and developed from the standpoint of a previously accepted speculative idealism of Hegelian type; and it suffered from the defects of the empirical study of religions in his day. Since then this empirical study has advanced enormously, and is providing us with the material upon the basis of which, without a pre-conceived Hegelian Idealism, a modern philosophy of religion may be tentatively constructed.

We are still far from free from this tendency to develop a philosophy of religion mainly from consideration of the principles of a previously accepted metaphysical theory apart from first considering the facts of religions. This is not the least evident among realist philosophers whose general attitude towards the empirical might lead us to expect something different. It is quite clear to the careful student of S. Alexander's *Space, Time, and Deity* (1920) that his treatment of deity and religion is not from an investigation into actual religion but a form of speculative continuance of his own metaphysical principles, supported by that from the realm of religion which appears to him illustrative of his contentions. And here perhaps is the place to make a protest against a not uncom-

mon tendency of philosophical writers who, while rightly contending that those who write on the philosophy of science should really know something about the actual physical sciences, do not take the trouble or even acknowledge the necessity of acquainting themselves with the empirical knowledge of religions before they venture to write philosophically upon religion.

The direction of the development of the philosophy of religion is in principle partly suggested by the title of the Chair of the Christian Philosophy of Religion established at Oxford within recent years, and occupied by Dr. C. C. J. Webb. It is not necessary to criticise here the conditions required of the person to be elected to that Chair. The idea of the foundation is legitimate and in the line of scientific and philosophical advance. For it suggests that the specific elements of the Christian religion are to form the empirical basis for philosophical examination. Such a procedure is methodologically quite sound, as long as scientific and philosophical freedom is maintained and the danger of falling into the old type of apologetics with foregone conclusions is avoided. But the adequate development of a Christian Philosophy of Religion requires a detailed consideration of the contents of Christianity, its practices and emotions, as well as its doctrines. Dr. W. R. Matthews' *Studies in Christian Philosophy* (1921) is limited to a speculative justification of certain fundamentals of Christian Theism, along somewhat traditional lines. The writer definitely limits himself to these fundamentals, but an adequate Christian philosophy of religion would include a philosophical examination of the nature and implications of the whole range of ideas in the Christian creeds and of the nature and significance of its rites and ceremonies, and of the character of its emotional dispositions, and all this as found in different variations in

the different Christian communities. There is, for example, a whole range of beliefs and practices concerned with sin and suffering, salvation and redemption, ignored by the earlier Natural Theology, which it would be difficult to bring under a few simple affirmations.

Analogous with studies of the Christian Philosophy of Religion with the empirical facts of Christianity as its empirical basis there might be a Buddhist Philosophy of Religion, an Islamic Philosophy of Religion, and so on, each religion forming the empirical basis for its own philosophy of religion. Such studies would indeed be valuable, and are almost a necessity. But none of them individually, or all of them taken grouped together could rightly be called the philosophy of religion. For while the philosophy of religion requires the consideration of historical religions it cannot be limited to any particular religion or religions, but must view all facts of religion as parts of a whole wealth of empirical data for its examination as details within the developments of that whole.

II.

To form an adequate conception of what is implied by a modern philosophy of religion it is necessary to understand the nature of philosophy as philosophy. There are various views as to this, and a complete defence of the choice of any one would involve a criticism of the others. Such criticism is not to be embarked upon in this paper. In a very general way philosophizing may be said to be a search for ultimate and complete truth, and a philosophy to be an approximation to or an embodiment of at least part of it.

Along with the notion of truth we have the notion of error. Let us observe what philosophizing seems to imply.

Either there is a distinction between truth and error, or there is no distinction. If we assume that there is no distinction between truth and error, then the two propositions: "There is a distinction between truth and error," and "There is no distinction between truth and error" must be at the same time both true and false. There is no way out of that impasse but to make the contrary assumption that there is a distinction between truth and error. This assumption seems inevitable for all philosophizing. And along with this there must be acceptance of the belief, faith, conviction, whatever one may call it, that the philosophizing mind has the capacity to distinguish between truth and error. If it is suggested that this is merely a theoretical belief, then the judgment whether this is a true or false belief can only be made if the mind has the capacity of so judging. We therefore here reach an ultimate, which, if there is to be any science or philosophy at all, must be accepted.

But the distinction of truth and error is a distinction between true propositions or judgments and false propositions or judgments, finally and ultimately, of propositions which are simple in form, that is, cannot be analysed into other and simpler propositions. All such ultimate propositions in themselves or as the basis of a course of argument or reasoning, themselves include terms which themselves are expressions of ultimates or of a complex of ultimates. The apprehension of the meaning of an ultimate proposition is not the same thing as the apprehension of an ultimate of which a term is the expression. In other words there may be experiences without the presence of judgments; at least there seems nothing adequate to make us reject this view. But there can be no judgments without the apprehension of ultimates such as are expressed by terms. A term which expresses an ultimate is said to

be indefinable. Such terms expressing ultimates are the real coin of all thorough-going philosophizing. They are fundamental. The omission of any such terms, or the assumption that there is only one ultimate term. makes for narrowness and inaccuracy in philosophy. There is a sense in which philosophy is a search for a unified intellectual scheme, but it must be one in which all ultimate differentiations have their place accorded to them. It is, therefore, of the utmost importance to examine each realm of experience and to acknowledge the ultimates of all kinds, religious if any, as well as those of the socalled physical and ethical.

Or the position may be stated otherwise. It is sometimes said by persons not familiar with philosophical discussion that religious truth is to be tested by reason. Such a statement is in the highest degree vague and indefinite. Reason does not function in vacuo. All reasoning if significant from the point of view of truth refers to facts of some kind. Though reasoning must conform with the principles of logic if it is to be accepted as valid. the fundamental matter besides conformity with these principles, and as important as such conformity, is the apprehension of the meaning of the premisses. And there is about the comprehension of ultimate premisses something immediate. Further, it is one thing to understand a proposition, and another to accept it as true or reject it as false. The mind in its capacity of distinguishing truth from error is capable of both these things. What is involved in the understanding and the accepting of a mathematical proposition as true is not the same as is involved in understanding and accepting as true a proposition of religious import, such for example as: The Other of religion is holy. The difference between the mathematical and the religious activities of the mind affects the char-

acter of the apprehension of the premisses and the conclusions in trains of thought in mathematics on the one hand and the philosophy of religion on the other.

The philosophy of religion insists that its subject must be treated in a manner similar to that adopted in other branches of knowledge, and that no more and no less should be expected from it than what corresponds with their attainments. Each science claims to have the validity of its propositions considered first in relation to its own realm of facts. The statements of biology are not to be tested primarily by reference to facts of astronomy, nor those of psychology by reference to the physiological. The validity of the propositions of the philosophy of religion is to be tested first by reference to the experiences of actual religion. If it be said that thus one is moving in a circle, the same is true for all sciences. Their propositions are arrived at by consideration of facts in their own spheres, and they return to facts in their own spheres to test the validity of their propositions. But though related the facts are not necessarily in both instances the same. The primary concern of each branch of knowledge is that its propositions shall be consistent among themselves and shall be comprehensive, that is, shall have reference to the whole realm of relevant facts. The same is true for the study of religion.

It is proposed to make a distinction between metaphysics and philosophy. By the former is meant the consideration of the ultimate concepts applying to reality in general apart from any specific empirical events considered as such. These ultimates constitute as it were the skeleton of reality. But they are not adequate to the details of the course of events as it is found in history and the details of the world as it lies before us. If, therefore,

metaphysics is concerned with ultimates, philosophy has to strive for a wider view of the whole, that is, of the ultimates in relation with empirical details. Philosophy is not simply the analytical discipline of metaphysics, but also a constructive endeavour such as the Germans call a *Weltanschauung* and *Lebensanschauung*.

As so described, a philosophy is the whole of a general view of ultimates and of the details of experience insofar as they may be expressed in concepts and propositions. As compared with such a whole a philosophy of religion must appear to be a partial view. In this it is similar to a number of other disciplines, as a philosophy of natural science (or of Nature), a philosophy of history, or a moral philosophy. In some respects these hold an intermediate position between a general philosophy and particular bodies of empirical knowledge, such as the natural sciences, history, ethics. The philosophy of religion holds a similar intermediate position between general philosophy and the organised body of knowledge of religion as empirically investigated.

The whole of knowledge in a philosophy may therefore be considered as though built up on the basis of bodies of empirical knowledge (as the socalled sciences), through departmental philosophies having particular relation with groups of such bodies of knowledge, thence to a general view taking up into itself these particular philosophies in the most comprehensive and systematic manner possible. It is a question for the decision of the individual thinker whether he shall include as a part of or an appendix to a departmental philosophy some consideration of its relation to a general philosophy into which it should fit. As a rule it may be regarded as an advantage rather than otherwise that it should be explicitly pointed out if a departmental

philosophy suggests a certain type of general philosophy more than any other.

A departmental philosophy is concerned in the first place solely with the empirical knowledge in the particular spheres of which it is the philosophy. It has two tasks in connection with this. On the metaphysical side it must investigate what are the ultimates of experience in these particular spheres and the concepts or terms which refer to them. But as it is not merely metaphysics, it has also the second task of obtaining a general view of the empirical details in relation with these ultimates, a view as comprehensive and systematic as possible. It may be said that a departmental philosophy has to seek the ultimate and complete truth (as far as may be) of the alleged knowledge of its particular sphere or spheres.

The reference to the concept of truth brings us to a consideration of the greatest importance. Truth (or error) can be affirmed only of judgments or propositions expressing judgments, and these indeed when reduced to their simplest forms. That is, the truth of a complex proposition depends upon the truths of the simple judgments into which it may be analysed. And the truth of simple judgments—which are at the basis of all forms of reasoning—can only be known in simple apprehension, by the capacity of the mind to distinguish truth and error which we were compelled to admit as fundamental for all philosophy. But this apprehension is an apprehension of ultimates (or what at any level of human mental development are accepted as ultimates) such as are expressed in terms which are indefinable.

The departmental philosophies are in one way or another dealing with spheres of knowledge which are distinct just in that they refer to different types of ultimates.

Thus the truth in a particular departmental philosophy can only be tested internally to itself, that is, by the particular forms of apprehension of its own ultimates. The truth of the propositions in a moral philosophy (unless they overstep their legitimate sphere) depends solely on the mind accepting them as true on the basis of its own apprehension of ultimates of a moral type. Once admit the existence of experiences which are distinctively moral, and the basis is admitted for a moral philosophy. And the same thing applies in one way or another, however complex the data may appear at first sight, for all departmental philosophies.

A genuine philosophy of religion is therefore possible only if there is a realm of experience with distinctive ultimates, or expressed otherwise, only if there is something distinctive which is meant by the name religion. Such a contention does not beg the question as to whether it is or is not a particular form or forms of subjective feelings giving rise to superstition. It is quite clear (as the existence of the name itself suggests) that human life as manifested in our own times and in history includes something connoted and denoted by this name. It is about this that the philosophy of religion represents a philosophizing. The philosophy of religion must be on a basis of the empirical study of religion as it appears and has appeared.

This is not the place to do more than to indicate briefly what the empirical investigation of religion implies. It includes the psychology of the religion of the individual as such and in his relationships in social groups. It includes also a survey of the facts of the course of religion (or religions) in history. In these studies there is in the former more emphasis on the analytic method; in the latter on the genetic—but not entirely so. An application of the

method of comparison is useful in promoting the systematization of this empirical data for the purposes of philosophical consideration.

The speculative bias of philosophy in the past has tended to direct attention chiefly, sometimes almost solely, to ideas and concepts. A modern philosophy of religion has to overcome the tendency to such a bias, which would lead it mainly to a consideration of the theoretical doctrines and beliefs of the religions. It has to take religion as it finds it—a complex in which emotions, practical attitudes and actions as well as beliefs are involved. Taking into account the many "varieties of religious experience," it will differ greatly from the earlier Natural Theology and forms of speculative Theism with their attention centered almost solely on the ideas of God, immortality, and freedom.

It is not possible at the outset to make an enumeration of the problems of a philosophy of religion, for these arise out of the great variety of details of empirical religions. The significance of these details in themselves and their consistency or otherwise amongst themselves have to be investigated. Nevertheless in the course of the development of a modern philosophy of religion, so far as that has yet gone, some leading problems have suggested themselves. There is, for example, the problem of the nature and sources of what is called religious knowledge. In this connection the nature of the characters and functions of prophets and saints, the significance of inspiration and revelation, and even of the earlier practices of divination, call for consideration. Other problems, which in their full treatment involve very much investigation of details, may be grouped around what might be called the subjective and the objective aspects of religion. The former have to do

with the nature of the human self as it is ultimately involved in religion. The latter have to do with the nature of the other than self which is ultimately involved in religion. And the discussion of these two groups of problems inevitably leads to the problems of the nature of the relations between the self and the other in religion.

A philosophy of religion which is developed in relation with religion, as it appears and has appeared in history, will be led to consider problems which from other points of view are too easily overlooked. There is, for example, the relation of man to Nature, and the place which Nature has taken and may take in religion. Nature played a very prominent part in early religion; and there are some thinkers today who would have us believe that religion can consist in nothing other than in man's relation to Nature. Or again, in line with the great neglect of Nature by some of the higher religions, there are those who find little in it of significance for religion. This whole subject raises serious problems which demand careful detailed consideration. The philosophy of religion finds in empirical religion not simply forms of conceptual understanding (or misunderstanding) of Nature, but also definite types of re-action to it. And the significance of these re-actions it has to inquire into. The formal more speculative endeavours which such a modern philosophy of religion replaces neglect these and give attention to formal conclusions to which natural science may lead with its purely intellectual aim. In his Gifford Lectures on *The Domain of Natural Science* (1922) E. W. Hobson seems almost to contend for a definite separation of religion and natural science. The only contribution to the philosophy of religion which he considers natural science able to make is its manifestation of order in Nature. That

of course, is important for any philosophy of religion. The early literatures of the great religions reveal a grasp of the same characteristic of Nature, and give it the place due to its importance. But the formal speculative endeavours stop, as far as Nature is concerned, with the acceptance of such a contribution from natural science. With its consideration of the re-actions of man to Nature as seen in the empirical religions, a modern philosophy of religion cannot stop at that point. For it sees emphatically presented that these re-actions are based not only on the apprehension of order in Nature, but also on the apprehension of kinds of discord and conflict. Empirical religion shows the attraction to Nature on account of some of its features, but it also shows a repulsion from it on account of other features which make it impossible for it to be an adequate object as a resting place for the human spirit.

The formal speculative endeavours which our modern philosophy of religion would replace have an attitude towards history analogous to that noticed with regard to Nature. They take into consideration those characteristics of existence which, as it might be put, make history possible, the fundamental principles, the underlying metaphysics of history. But the modern philosophy of religion finds that attitude inadequate, although it also must give due place to consideration of these fundamental principles, the underlying metaphysics. For, concerned with empirical religions, it finds that these affirm definite relations of religion to particular persons and events in history, and to the general course of empirical history. It is for the philosophy of religion to inquire into the significance of these elements in religions. But this consideration of the historical must be in relation with the actual details which

the religions have concerned themselves with. The late E. Troeltsch had a very keen appreciation of the importance of the historical for the study of religion, yet he did not seem to get free from the speculative attitude and a somewhat Hegelian conception of sequences of culture epochs, as the only significant feature involved. The philosophy of religion ought to endeavour to arrive at a conception of the historical from its point of view, and one important factor in the historical is the development of religion itself.

It is not merely actual historical events deemed of importance by historical religions which call for consideration by the philosophy of religion with reference to the significance which is given these events, and whether that significance is justified. It is also important for it to take note of the myths and legends which have grown up within religions. For, though it may be clear that these are not literally true, they may nevertheless be expressions—often of a poetic order—of experiences which themselves have a real import and significance. The frequent origin, or the continued acceptance of certain common types of myths and legends, may not unreasonably suggest that they are attempts to express characteristics of experience and reality.

And in this connection it should be observed that the empirical data for the consideration of a modern philosophy of religion is not to be sought entirely in definitely organised historical religion. Literature, quite apart from the socalled sacred scriptures, contains very important material concerning those attitudes and types of experience of the human mind which are usually classed as religion, even when the terminology used is often quite other than that used in the religions.

The modern philosophy of religion does not call for a definition of religion in order to define its limits before it begins. Religion needs at the outset to be understood in such a general sense as not to exclude anything which at any stage in its history appears to have been accepted as part of it. That any such constituents have at later times been definitely discarded is itself a fact of importance for the study. One thing, however, seems essential for the modern philosophy of religion: it recognizes religion as including types of practical re-actions and of emotional dispositions and attitudes. It cannot therefore place its sole attention upon the ideas expressed in the religions, but must enquire into the nature of these re-actions and of their results.

It should be accepted as a definite part of the modern philosophy of religion to consider the relation which its conclusions on the basis of its own data bear to the conclusions of other departmental philosophies on the basis of their own data. Thus, from its own point of view, it should indicate the type of general philosophy which is suggested. With this, however, the step is made to the standpoint and tasks of general philosophy. But, analogous with this, it does not seem inappropriate that the philosophy of religion should consider the relation of religion as a distinct practical attitude to the other practical attitudes and activities of life, noting the character of any discordance, and inquiring whether the claim frequently made that life as a whole may be harmonized from the standpoint of religion is justified.

A philosophy of religion such as is here suggested should be able to estimate to some extent the validity of the claims made for the different religions and in some measure to bring the different religions into a reasonable relation.

The apparent or real contradictions between the doctrines of the different religions, and between different sects, suggest the need of an independent philosophy of religion. It is evident that this demands balanced judgment on the part of the philosopher of religion; but the difficulty of this is no justification for regarding it as impossible. It is wrong to suggest that this involves, in the words of Dean Inge, being "an honorary member of all religions and a true adherent of none." Difficult though it is, it is nevertheless possible for a man to admit the values in other religions than his own, and to recognise the errors and defects in his own, without separating himself from the religious life of the community of which he consciously calls himself an adherent.

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THE NATURE OF SYSTEMS

Part II.

A MATHEMATICAL system is any formal system which is not a logic. Its postulates are non-tautological. Its assumptions are arbitrarily true, or as the term indicates, are assumed to be true. A mathematics states propositions which could be otherwise; if one of the propositions were denied an equally significant mathematical proposition would be thereby created. On the other hand the contradictory of a logical proposition must be false, because the denial of the truth of a necessary truth is necessarily false. This is the meaning of necessary truth; it is such that it is impossible that it be false; its denial therefore cannot possibly be true.

Like logical primitive propositions, the postulates of a mathematical system are premisses which are connected with the theorems by the 'correlator' entailment. Conjoined with the theorems they represent tautologies; the union of the postulate with the theorem is analytically true. But the postulate itself is not analytically true. "P v Q" is not a logical proposition, though it may be a mathematical one. A logical proposition would be "P v Q entails P v Q." If a postulate of mathematics were "P v Q" the assertion of it together with a theorem would yield a proposition equivalent to or identical with "P v Q entails P v Q," while a similar union of a primitive proposition with a

theorem in logic would yield "'P v Q entails P v Q" entails "P v Q entails P v Q.'" The mathematical postulates and theorems are not tautological, but they are connected tautologically; while the logical primitive propositions and theorems are all tautological in addition to being connected tautologically.

There are three current interpretations of the relationship of mathematics to logic. The present interpretation has points of agreement with each, though it has a number of vital differences. Our doctrine of "systematic formalism" grants that logic and mathematics can both be systematized, denies that either of them has a special content, and insists that their difference is to be found in the nature of the significance of their units.

Most symbolic logicians view mathematics as a part of logic. Its chief protagonist has been Russell, and with some minor differences is a view which is shared by Whitehead, Couturat, Wittgenstein, Ramsey and Lewis. "Pure mathematics is the class of all propositions of the form 'p implies q' where p and q are propositions containing one or more variables, the same in the two propositions, and neither p nor q contains any constants except logical constants."[19] "The fact that all mathematics is symbolic logic is one of the greatest discoveries of our age; and when this fact has been established, the remainder of the principles of mathematics consists in the analysis of symbolic logic itself."[20] ". . . all mathematics is deduction by logical principles from logical principles. . . ."[21]

Though we acknowledge that the theorems of mathematics are deduced *by means of* logical principles, we have been unable to discover the logical principles *from which* they are deduced. The postulates of mathematics

[19] B. Russell, *Principles of Mathematics*, p. 3.
[20] Ibid., p. 5.
[21] Ibid., p. 5.

as Langford has pointed out[22] select one of a given number of alternatives. Logic states the totality of these alternatives; mathematics isolates one or more of them, and thus by arbitrarily excluding some possibilities, says something more than a tautology. As the union of a postulate and theorem is a tautology, and can be found in a logic, the postulates and theorems of mathematics can be viewed as significant fragments of a tautological proposition, and thus as derivable from logic, by analysis. But this is not deduction. Only if we take the postulates and theorems together do we get a logical proposition, and it is only then that we can deduce logical propositions (i. e. further combinations of postulates and theorems) from logical propositions.

So far as I have been able to understand Wittgenstein's cryptic remarks, our views regarding the nature of logical propositions in many respects are almost identical:[24] "In logic there cannot be a more general and a more special" (5.454); "In logic, process and result are equivalent" (6.1261); "Proof in logic is only a mechanical expedient to facilitate recognition of tautology, where it is compyicated." (6.1262); "All propositions of logic are of equal rank; there are not some which are essentially primitive and others deduced from these." (6.127). In 6.2 he holds that "Mathematics are equations and therefore pseudo propositions." "The logic of the world which the propositions of logic show in tautologies, mathematics shows in equations." (6.22)

Only Wittgenstein will be able to tell whether his description of mathematics as a logical method, or as a method of logic (6.234) is the same as what we have meant by saying that mathematics employs tautological

[22] See "Analytic Completeness of Sets of Postulates," *London Math. Soc.*, Vol. 25, p. 17.

[24] See *Tractatus-Logico-Philosophicus.*

principles by which to derive its theorems. The interesting point is the interpretation of mathematical propositions as equations. They can be taken as equations in two ways. One can read a postulate of the form "P v Q" as the equation "P and Q are two independent elements of class K equals P v Q," or as "P v Q" equals "—P implies Q." In both cases the equation amounts to the statement of a voluntary definition. One can take a different definition and secure a different system. If we contrast these two with similar statements in a logic, as "'P and Q are two independent elements of class K" equals "P v Q entails P v Q'" or "'P v Q entails P v Q" equals —"P implies Q" entails —P implies Q,'" the same difference between non-tautology and tautology will become evident, though not as different sets of symbols to express the same thing; but in one it will be an equation whose equates are deniable and in the other it will be an equation whose equates are tautologies. It seems that the equations of which Wittgenstein speaks are those of the form "'P v Q equals "—P implies Q.'" Undoubtedly every proposition can be put in this form, be it mathematical or not. Not only does Wittgenstein's interpretation require that the theory of numbers be limited to finite numbers, but it makes it impossible to understand how mathematics could ever require ingenuity. If mathematics "expressed no thoughts" and was a mere concatenation of meaningless equations, every bit of the science, from top to bottom, could be ground out by a machine. The important thing is not the equations, but the equates, and all the work has already been done when the equates have been uncovered.

The intuitionistic mathematicians, of which Brouwer is possibly the best known, contend that instead of mathematics being a part of logic, logic in fact is derived from mathematics. Brouwer, for example, holds that logical

laws, like the law of the excluded middle are derived from a study of finite series. When we come to infinite series, this law is neither applicable nor permissible, ". . .der Satz vom angeschlossenen Dritten ein unerlaubtes mathematisches Beweismittel sei." Brouwer, however, has not shown that this or any other logical law is demonstrably derivable from mathematics, so that his contention amounts to the questionable and somewhat irrelevant statement that logic is historically subsequent to mathematics. As to the inapplicability or irrelevancy of the law of the excluded middle to infinite series, this is no more or less true than its inapplicability to finite series. The law always applies if one wishes to divide any group into two classes, but if one wishes to make more than two classes, then some other value of the generalized law of the "excluded n + 1" is relevant. This point will be examined a little more in detail in section 14, though it may be advisable to remark now that the "logical" difference between finite and infinite series for Brouwer comes down to the ability and inability to discover the concepts which are incompatible. There is, however, an important and neglected sense in which mathematics is prior to logic. The development of systems involves the employment of the concept of a class. This concept is used to make possible the systematic treatment of propositions and classes derivable from the analysis of them. If class be taken as a concept of mathematics—at least of some general system of mathematics—the derivation of the concepts of mathematics from those of logic is accomplished by employing some concepts of mathematics in the first place.

That logic and mathematics are independent and different sciences is the most popular view. Historically it is first, and today it is being defended on all sorts of

grounds by men with comparatively little knowledge of symbolic logic.

One of the favorite arguments is based on a Kantian distinction between synthetic and analytic judgments. We have held that logic does deal with analytic propositions and that mathematics states significant (and if it does not lead to confusion) synthetic ones. But they are not synthetic in the sense in which Kant sometimes took them to be. It is only the postulate or the theorem apart from demonstration and interrelationship which are synthetic, or at least non-tautological. Mathematics is arbitrarily true, insofar as one considers the postulates or theorems separately and apart from demonstration; it is analytically and necessarily true insofar as one considers the theorems as following from the postulates.

XI

The unit correlator in the logical proposition is "entailment." It always determines a tautology and has therefore been designated as a tautological correlator. The disjunction which arises from denying the antecedent in such a tautological unit, is also a tautological correlator. It is distinguished from ordinary disjunction by the fact that both disjuncts cannot be false. '—p : v—p' has a different correlator than 'p v q.' Both these correlators (there may be others definable in terms of these), are repetitive, so that a tautological unit may be tautologically correlated by the same tautological correlator with another tautological unit. Other correlators in logic are compound repetitive correlators. They always determine, in logic, a significant compound, and never a tautological unit. Despite the fact that G. E. Moore pointed out many years ago the difference between implication and entailment,

Russell and his followers still persist in confusing a compound with a unit correlator. These compound correlators are the so-called logical constants used to determine *units* in mathematics. This is an obvious consequence of the fact that the compound correlators determine elements which are tautologically correlated, and that mathematics is interested in tautologically deriving non-tautological propositions tautologically from other non-tautological propositions. Anything can be symbolized in terms of logical constants, so that the mathematical assertion uses as a unit correlator a compound correlator of a logic. When used outside of logic, these compound correlators will be referred to as material correlators.

Wittgenstein takes all logical correlators, unit and compound, as signs of punctuation, (5.4611). They have no existential counterpart—"My fundamental thought is that the "logical constants" do not represent" (4.0312). Russell takes this to mean that "logical constants are not represented by signs, but are themselves present in the proposition as in the fact" (p. 11). In what sense are signs of punctuation present in a fact? But even if this be a true interpretation, the remark has little importance for formal logic, being a proposition in a metaphysics. It says something about the character of a correlator as capable of existing in the world.

There may be "ors" and "ands" in the universe. "And," in fact, seems to me to be as real as any relation I know, and there seems to be no good reason for denying that it is a sign of a form of an existential compound element. Be that as it may, symbolically and formally, the material correlators determine units in mathematics, the logical compound correlators determine complexes or compounds in logic, and the tautological correlators de-

termine logical units. Mathematical units are usually made up of compounds which contain an undescribed compound correlator, whose functional properties are revealed in terms of the material correlators. The totality of the postulates expresses the functional properties of this correlator and it in turn determines the significance of the elements.

From this point of view logic and mathematics may be distinguished on the ground that the former determines the functional significance of logical compound correlators by means of tautological correlators, and the latter determines the functional significance of non-logical or material compound correlators by means of material ones. (This point will never be recognized as long as no distinction is made between implication and entailment.)[25]

XII

A postulate is an assertion whose truth is undemonstrated but not necessarily undemonstrable. It is the premiss of a tautological proposition, whose consequent is a theorem. It is to be distinguished from a hypothesis and a definition. A hypothesis is a proposition which conforms to certain methodological conditions. It is capable of experiential denial. It states something about fact, and should fact deny it, must be abandoned. A postulate of a formal system, on the other hand has no reference to fact, and a postulate of an existential system is asserted independently of the state of the facts. The postulate, in short has truth only by courtesy or special endowment. No fact can deny it, for no fact is pertinent to it. A

[25] Implication, which is a compound repetitive correlator in logic and a repetitive unit correlator in mathematics, is identifiable only so far as the system itself is identified as a logic or mathematics. Mathematics may use implication to determine compounds of compounds, but these are further determined in units by implication as well; logic never uses implication to determine units.

postulate of mathematics may serve as a hypothesis in science or engineering, but whatever service it may render there can neither help nor hurt its postulational status. The logical primitive proposition is necessarily true. Its validity can be ascertained without recourse to experience. As has been indicated, it is not needed in order to substantiate another logical proposition, for every other logical proposition is also analytically and necessarily true.

The theorems in a mathematical system are true because of the postulates. The postulates entail them. They are not necessarily materially equivalent with the postulates, so that the denial of one of the postulates will not necessarily involve the denial of the theorem. Such a denial of a postulate may foreclose the possibility of that theorem having a place in the system, but any given theorem may follow from more than one conjunction of postulates. In a logical system, no one theorem can be supposed false without the tacit supposition of all the postulates and other theorems being false, for they are all true for the same reasons. Any proposition in logic is materially equivalent with any other, be it primitive or theorem; no proposition in mathematics is necessarily materially equivalent with any other. As all true propositions are materially equivalent, the postulates are materially equivalent by hypothesis. The proofs of independence, etc., to be discussed shortly involve the supposition that one of the postulates, at least, is false. But once they have been assumed to be true, it follows that on that supposition they are materially equivalent. (*5.1 $P \cdot Q \cdot \supset \cdot P \equiv Q$).

Definitions are of two kinds—verbal and "intensional." The former arbitrarily states that certain symbols are to be used in place of others. If we decide to use the word

"dog" where we have formerly used the word "cat," our decision involves nothing more than the substitution of certain marks, sounds or indices to serve a purpose for which some other has been or is employed. The definitions in systems are all intended in this sense, though most of the definitions in the Principia Mathematica are highly significant, which accounts for the miraculous, deduction of mathematics from logic, made there. They are the results of a volition and are neither true nor false—though they may effect the understanding of truths and falsities. My decision to tell a lie is itself neither a lie nor a truth. It may help to reduce the totality of my possessible truths, just as an arbitrary exchange in the use of conventional signs may lead to confusion. Both the decision and the exchange are events, of which the definition is the symbolic expression. The act of deciding is the fact of equating; the result of the decision is the equation. By correlating one complete definition with another, one can symbolize the significance of different decisions. This will not necessarily be a correlation of symbols representing something, but is always a correlation of facts, which happen to be symbols.

It is also possible to view a definition as a propositional function, by the simple expedient of taking it as a proposition about the future (use of symbols.) There are some grounds for believing with Broad that assertions about the future have no objects. They may be said to be propositional functions for whose ambiguous constituents definite values are still to be provided. Such a view would make it impossible to be in error about the future, and this is a serious limitation. In another place we shall advance an interpretation of assertions about the future, in which some middle road will be sought between the analysis of

them as propositions and as propositional functions. But if one desires to accept an interpretation of assertions about the future as propositional functions, there will be no great difficulty in taking a definition as a propositional function which becomes a proposition after a systematic exposition has been completed—for such an exposition would reveal whether we had actually used these different symbols interchangeably.

Any one of these interpretations makes it still more apparent why mathematics cannot be reduced to formal logic. Taken as a system of equations, it is a system of volitions or propositional functions, in neither case being true or false.

"Intensional" definitions attempt to express the unique essence of a term. Despite the great interest in such definitions since Aristotle, very little has been done, to show the essence of the essential definition. There is no doubt but that there is a definition which is not verbal. G. E. Moore's indefinable good is not verbally indefinable for any number of different symbols can be substituted for it. As he views it, it is indefinable because it has no parts, and because its understanding or perception does not require the intermediation of any other concepts or percepts. Whether good has this character is not a question to be discussed here; but it is in point to note that there are entities which are definable in the sense that they have parts or have their character expressed by making them part of something else. A definition of such an entity would involve the statement of all its parts, or the description of its significance. But the entity is not found by noting all its parts; counting the bricks will not yield the house. It is the totality of these parts, as a totality, that expresses the character of the whole. And the nature of this totality is

found by the discovery of their significance in this whole; i. e. by understanding the interrelationship of the different parts. Definition, then, reduces to the statement of the significance of parts of a given whole, or of a given part in a larger whole. This is the sense in which some mathematicians have described postulates as "definitions." As nothing is known about the character of the elements in a formal system beyond their significance, they are to be characterized only in terms of it. But in this sense even the indefinables are defined, for they are parts of some significant whole. The only alternative left is to consider the indefinables as the systematically or existentially simple. They have no parts, and thus no internal significance; while that which is intensionally definable has parts and when defined is stated as equivalent with the totality of its parts in all their interrelationships.

XIII

The heart of systematic development lies in the principles of substitution. There need be no inferential rules, but there must be rules of substitution. Without substitution there can be no inference, for the inferential rule is applied on a complex unit created by making substitutions in a postulate or primitive proposition; while if there are only substitutional principles, theorems can be developed simply by making the exchange of one element or compound for another in accordance with the rule to be shortly stated.

Substitution within a system is a tautological operation. Were it not, there would be no reason for supposing that the result would be a unit which had the same grounds of validity that it had had before the substitution had taken place. It could then allow the derivation of non-tautological

propositions from tautological ones (i. e. possibly false from necessarily true propositions); tautological propositions from non-tautological ones (i. e. necessarily true from possibly false propositions) and false propositions from true ones (i. e. self-contradictory propositions from tautologies or postulationally invalid propositions from postulationally true ones.)

Each system requires definite laws of substitution, but in each case these rules are illustrations of the principle: *a substitution must involve no change in the significance of a unit.* The various applications of this rule have been ably discussed by Eaton in the chapter on Formal Deduction in "Symbolism and Truth," so that there will be little point in going into this matter in detail. We shall deal only with a number of basic points which have so far been neglected.

In a formal proposition containing one element, the substitution of another for it wherever it occurs does not yield a new proposition. There is no difference between "P entails P," "—P entails —P," "Q entails Q" and "—Q entails —Q." The elements in each case have the same significance and are capable of the same range of values. Whatever differences these elements have is to be determined only in terms of one another, and cannot be found by describing their function in the above units. —P can stand for true propositions or false ones just as P can, and it may represent *any* proposition just as P may, though should they both appear in one assertion, the supposition that one is true will involve the supposition that the other is false.

Compounds are to be substituted for elements only in homogeneous systems. It is only when the correlation of simple elements yields an element of the same class as

these simples, that they can be correlated in the same way as these simples were. It is in such a case that the presence of a repetitive correlator is obvious. The simples, if they take as values *any* entity having a specified property, are more general than any compound which represents one significant argument to that property, and cannot as a rule be substituted for them. Consider the proposition:

1. P entails that Q implies P.

This is a tautology. If in this proposition we substitute "Q" or "—P" for "Q implies P" we would in one case get a false proposition (or if entails were then taken as implies, a possibly false proposition), and in the other, a necessarily false proposition, so that it appears that simple elements cannot be indiscriminately substituted for compounds. Yet in any case where a simple element occurs, a compound can be substituted for it. By substituting "Q implies P" for "P" in the above proposition we get:

2. Q implies P entails that Q implies that Q implies P, which is also a tautology. It seems then that compounds can be substituted for simples, but simples cannot be substituted for compounds. Does this mean that a compound is a value of a simple? If it does, then it is not true that all the propositions in logic are on the same level; that there is no more general and less general in logic, for the propositions of the form 2 would be less general than those of form 1. As a matter of fact in any given unit it is possible to discover a simple element which can be substituted for a compound, so as to yield a tautology. If we substitute P for 'Q implies P' in No. 1, we get the tautology P entails P, or more indirectly, if we substitute in No. 1. 'P implies Q' for Q, we get the proposition:

3. P entails that P's implying Q implies P.

This proposition which has been achieved by substituting a compound for a simple is equivalent with

4. P entails P.

There is no distinction between these propositions, except in symbolic form, so that No. 1 instead of having values whose employment make the proposition less general than before is capable of taking values which make the proposition as general as before. Yet the fact remains that any compound at all may be substituted for any simple element, while not all simples can be substituted for a given compound, so that in one sense the compounds must be less general than the simples.

The reason for this paradoxical state of affairs is that a compound may say something significant about its members, which significance is lost by the substitution of the simple for it, so that the entailment which is based on this significance may then be unjustified. The significance of a simple is determined by its place in a compound or unit, and the substitution of a compound for this simple gives the compound the same significance that the simple had. The substitution of a simple for a compound, however, may modify the structure of the proposition and determine the simple to have a different significance from the compound. In No. 1, "Q implies P" is a function of the elements and the correlator; the substitution of —P for this compound eliminates that functional significance and —P is determined simply by the unit correlator, so that in place of a unit made up of simples having a definite relation in a compound, we get a unit whose elements may not have the same "truth grounds"[26] as the original unit.

Similar remarks apply to the intersubstitution of units with simples and compounds. In logic the units are an-

[26] Wittgenstein, *op. cit.*, 5. 101.

alytically true; in other systems they are true by postulation or by entailment. The substitution of an element or compound for these units will, as a general rule, involve the substitution of that which is possibly false or that which has no significance for that which is necessarily true because of the nature of its significance. However, in any unit, a unit may be substituted for the elements contained in it, providing the system is homogenous; i. e. if the unit is itself an element of the same class as the constituent simples. In a general logic, where the elements, compounds and units are all propositions, the compounds and units can always be correlated in terms of other compounds and units for the very same reason and in the very same way as the simple elements are. However, if the elements are arbitrarily determined to be propositions of one kind, and the units and compounds are propositions of another, such substitutions cannot be made. If the first five numbers of the *Principia Mathematica* are intended to apply only to elementary propositions, the substitution of general propositions for the simples, must be invalid. The elements in these numbers, if the system be taken as general, must be real variables symbolizing *any* proposition, and if the correction in the second edition be accepted, and apparent variables be added to every real variable, these propositions must, contrary to the theory of types, say something about *all* propositions.

An entailment involves the correlation of an entailor with an entailee.[27] Whatever alternatives the entailor affirms, must be affirmed by the entailee. This gives us two general cases: 1. the entailee states just the alternatives stated by the entailor, and 2. the entailee includes these alternatives within a larger set. It should now be

[27] In all these instances the definition of implication and the corresponding definition of entailment are those used in the Principia Mathematica, though the distinction between implication and entailment is not made there.

evident why every simple, compound and unit tautologically entails a tautology, and why tautologies can only entail tautologies. A tautology represents an exhaustive set of alternatives, and thus includes the alternatives represented by every element and unit. Nothing but a tautology can contain the alternatives represented by another tautology, so that the entailee of a tautology must be a tautology.

The first case is obviously a tautology, involving a repetition in the entailee of what had been stated in the entailor. "P entails P," "P implies Q entails —P or Q," "P entails that —P v Q implies P" " 'P entails P v Q' entails that 'P entails P v P • Q' " are all in this group. The last illustration represents a tautology entailing a tautology—the way in which all tautologies can be correlated. Such entailments are, of course, symmetrical; either term can be the entailor which entails the other as entailee.

That the members of the second group are tautologies is a little more difficult to see, due to the fact that the entailee adds alternatives to those stated by the entailor. However, the addition of alternatives is not an addition in information and definiteness but a decrease. The entailee says less than the entailor. To know that it is raining, to take a famous illustration, is to know more than to know that it is raining or not raining. The second, the entailee, is true if the first is; but the entailor may not be true if the entailee holds. Such entailments, obviously, are not symmetrical, or to put it another way, though it is possible to go from the more definite to the less definite, it is not possible to go from the less to the more definite. The entailee says less because it definitely excludes less; the entailor says more because it definitely excludes more. If we deny an entailor and convert the entailment into a disjunction, it

will be apparent that the negative of an entailor states more alternatives than the entailee. The whole situation thus comes down to a progress from more to less definiteness, and may be best illustrated by the two propositions: "The book is on the table in this room" and "The book is in this room." The latter allows more possibilities than the former, but the former, for that very reason says more. "P entails Q v P," "P entails that P implies P" "P implies Q and Q implies R entails that P implies R" (the syllogism), and "P • Q entails P" are in this group. The last does not seem to belong in this group, for the entailee seems to be contained within the entailor. But this is an illusion due to the simplicity of symbolic statement. "P" says the same thing as "P • Q v P • —Q," i. e. if P holds, then either P • Q holds or P • —Q holds and v./v. "P" is an eliptical representation of this disjunction of conjunctions—a shorthand symbol for it. The disjunction indicates that the affirmation of P is made independently of the truth or falsity of Q, and that no matter whether Q be true or false, P is true; the simple affirmation of P indicates that the truth or falsity of no other proposition is relevant, i. e. whether Q be true or false, P is true. "P • Q entails P" therefore can be read as "P • Q entails P • Q v P • —Q," and the inclusion of the entailor in the set of alternatives stated by the entailee then becomes apparent.

In view of these considerations it is debatable whether the *Principia Mathematica* requires its inferential rule: *1.1 Anything implied by a true (elementary) proposition is true,[28] which is used "whenever we have to deduce a *proposition* from a *proposition*."[29] That a tautology entails a tautology is a tautological consequence from the

[28] This should be read as: Anything entailed by a tautology is a tautology.

[29] P. 95, second edition.

tautological definition of a tautology. As a systematic logician is interested in stating tautologies, he can take any consequent of a tautology and state it independently of the tautology which entailed it, without requiring a special rule to assure him that it is a tautology, for the nature of the entailor will assure that.

Where the inferential rule is important is in completely uninterpreted and in mathematical systems. In his analysis of the uninterpreted system, Eaton points out that the inferential rule can be interpreted as a possibility of substitution: "If p implies q in an uninterpreted system, this implication means simply that q can replace p *as a conclusion* in any deduction. Any series of transformations that leads to p leads also to q. . . . . If q is an "implicational substitute" for p, this provision does not allow q to take the place of p in any complex in which p appears; but it does allow q to be substituted for p when p appears at the end of a chain of deductions. . . . An implicational substitute for any expression cannot replace this expression as a premise, or as a constituent in any expression whatsoever, but it can replace it as a conclusion."[30]

This restriction on substitution is a necessary consequence of what we have said about the character of the entailee in the second case. The entailor being symmetrical with its entailee in the first and asymmetrical with its entailee in the second, the substitution of the entailee for the entailor as a premiss, may involve a non-tautological correlation. P entails both 'P' and 'P v Q,' but 'P v Q' does not entail P. To substitute P v Q for P as premiss may make P v Q entail what it cannot.

But why can a consequent replace its antecedent wherever that antecedent appears as a consequent? The answer should be apparent from the preceding analysis. Z entails

[30] *Symbolism and Truth*, p. 233.

an element X because X states the alternatives which Z states. Z is entailed by Y, because Z states the alternatives which Y states. By reason of the transitivity of equality, X must state the alternatives that Y does, and therefore must be entailed by Y, so that it can replace Z wherever Z is entailed. The same reasoning, but with different arguments applies when the correlator is implication instead of entailment. Z implies X because X is true when Z is true; Z is implied by Y because Z is true when Y is true. X must then be true when Y is, must be implied by Y and thus can replace Z wherever Z is implied. This reasoning obviously uses the principle of the syllogism, to justify inference which is itself a result of the application of the principle of the syllogism.

"The use of a general principle of deduction, such as either form of 'Syll.' in a proof is different from the use of the particular premisses to which the principle of deduction is applied. The principle of deduction gives the general rule according to which the inference is made, but is itself not a premiss in the inference. If we treated it as a premiss, we should need either it or some other general rule to enable us to infer the desired conclusion, and thus we should gradually acquire an increasing accumulation of premisses without ever being able to make any inference."[31] This is no more than to say that logical propositions must be assumed in order to justify the deduction of logical propositions from logical propositions—this, of course, providing it be necessary to secure logical propositions or tautologies through the medium of substitution and systematic inference.

The theorems of mathematics are either direct entailees of the postulates or are derived by making substitutions in the postulates or theorems which have already been as-

[31] *Principia Mathematica*, Second Edition, p. 106.

serted as direct entailees, and then applying the rule of implicational substitutes. The first form of substitution has been shown to be tautological; the second must be for it is demonstrated to be an instance of the syllogism, which is itself a tautology. Wherever the asserted proposition contains a previously acknowledged entailor, the nature of the correlation (entailment) allows the assertion of the entailee apart from this entailor, for the entailee is true for the very same reasons that the entailor is true, and the ground which allowed the assertion of one will allow the assertion of the other. Inference is then a matter of dissolving an entailment and asserting entailee apart from its tautological correlation with the entailor. The entailor as postulate being arbitrarily true the entailed theorem is true as a tautological consequence of that which is true. And if the entailor is a theorem, its entailee is true, because by the syllogism it is a tautological consequence of some arbitrarily true postulate.

XIV

All parts of systems, as well as systems themselves are subject to the three "laws of thought." Here again laws of logic are assumed before the statement of the laws of logic is possible. This is not to say that the laws of logic are themselves subject to the laws of logic, but that any *statement* of them must be so subject.

No physical mark, or for that matter, no existent retains its physical identity entirely in all places and at all times. What remains the same is its non-physical aspect—its character. One persists in using the 'same' symbol and when it is given a meaning, it is said to retain that meaning in all the different contexts. Despite the microscopic typographical differences in the Ps and Qs throughout

the Principia Mathematica, there is no difference in the respective characters of these symbols. The law of identity is assumed to hold, not only in definitions and to justify substitutions, but also to guarantee the permanency of what we are talking about. Here most certainly, consistency in discourse is a consequent of persistency in intent—which is essentially the meaning of the law of identity in actual practice.

The law of contradiction is assumed in all the fundamental discriminations in the system. That which is an element cannot at the same time be a correlator, nor can a correlator at once be an element in a given assertion. The correlators and elements in turn are discriminated one from the other on the basis of the predicates of the sub-classes to which they belong. And in each class the individual members are unique and exclude all others. That which is distinct must always be distinct and be excluded by that which is other than it. It was the employment of the law of contradiction which illustrated the advisability of the introduction of the relation-function to take the place of the so-called relations, which were supposed to have mutually exclusive properties.[32] In general logic, the definition of truth values is based on the acknowledgment of the law of contradiction, for the truth-values truth and falsehood are such that no element can have both. In all these cases it is the definiteness of characterizations which allows the possibility of significance.

The law of excluded middle is employed when we determine elements and correlators to be either in a system or not in it, when we determine each member of B to be either a member of K or of C, i. e. to be either an element

[32] The two kinds of entailees indicates that there are two kinds of repetitive correlators "entailment." Pursuing the analysis we made in connection with relation-functions, "entailment" without qualification is to be taken as a *repetitive-correlator function.*

or a correlator, and when we determine every system to be constituted by tautological or significant units. It is this law which determines each unit to be either a postulate (or primitive proposition) or theorem, and which is the basis for the contradiction in application, etc., between the different systems, discussed in section eight. It is this law which makes it possible for every concept to have an application either directly or indirectly, i. e. by affirmation or negation. It is here that division determines the possibility of characterization.

The law of identity is the law of fundamental unity. What it determines is a whole, even though that whole be capable of analysis which will reveal it to be internally disparate, and a multiplicity. The object of the law of identity is a logical simple, and its universal application means that everything can be taken as such a simple, no matter how many additional determinations are imposed on it, and no matter how complex it may actually be. Even a symbol of the self-contradictory is subject to this law, for as such a symbol it is taken as one. 'Round-Square' is always identical with 'round-square.' This is not the place to discuss the metaphysical statue of self-contradictory objects. If they are only incomplete symbols, as such symbols they retain a core of permanent identity; if they are concepts, they retain it as concepts.

Contradiction makes possible the development of different and independent systems. To have meaning or significance means not to have all meaning or all significance. That which was everything could not be distinguished from that which was nothing; the all-meaningful has nothing to distinguish it from the unintelligible. ("Meaningless" of course has meaning, but what is meaningless has none; that is the meaning of "meaningless.") Tautologies

are entailed by their contradictories and the null class is included in all classes because they are distinguished from one another in definite ways, not because they are identical. It seems that the principle of polarity is basic to meaning and intelligibility. To be is to be excluded; that which is, is what it is because something else is other than it and at once excludes it and makes it possible. The north pole, to take M. R. Cohen's example, is the north because there is a south, and were one eliminated or identified with the other, it could not be.

Eaton has contended that the law of the excluded middle "is not a necessity of thought in the same sense as are the principles of identity and contradiction in their most general forms. The latter are needed if concepts or symbolic expressions are employed in any way whatsoever; the former enters only when truth and falsity (as well as identity and distinctness of concepts) are considered. It is necessary for inference—for the deriviation of the truth or falsity of propositions from the truth or falsity of others; but it is not necessary for the very existence of meaning, as are the first two laws of thought."[33] Brouwer's attack on this law, though originating from a different ground, is essentially of this form. He too seems to believe that the law states that something must be either true or false, and that therefore it has application only in certain fields, e. g. in connection with finite series and not with infinite series.

The law of the excluded middle need not be restricted to truth and falsity. It simply says that something must have one determination, or must be determined by that which is excluded by such a determination. Truth and falsity are only particular cases of determinations, and the law applies where these notions are not applicable, e. g.

[33] *Ibid.*, p. 209.

where judgment has not taken place. In addition, it is possible to subdivide the excluded class, which is infinite, and determine an element as a member of one of the subdivisions rather than another. Thus we can say that every proposition is either true or not true. That which is not true is either meaningless or false. That which is false is either impossible or a contingent fact and that which is meaningless may be without form or without content,etc. It is possible therefore to maintain that a proposition does not conform to the law of the excluded middle merely because it conforms to the law of the excluded fourth. Perhaps it would be best to view this third "law of thought" as the law of the 'excluded "n + 1" ' This is a propositional function, which becomes a definite proposition when the number of possible determinations have been decided. If every human is a man, woman or child, then any H is to be characterized in terms of the triple exclusive disjunction; M:v:W:v:C. In any case, however, any two groups can be reduced to one, by taking them together as the contradictory of the other, so that the law of excluded middle can always find application, even though another value of the excluded n + 1 was first applied.

Is this law not necessary for 'the very existence of meaning'? It must be if meanings are related as polar opposites. The denial of this law is identical with the denial of the law of contradiction [—(P : v : — P) ≡ —P. P,] and the law of contradiction, even for Eaton, is valid and necessary in the realm of meanings.

XV

Different logical systems are created, not by denying a primitive proposition and setting the contradictory in its place, but by adding additional primitive propositions to a

minimal set, or by originating different forms. Nothing is asked of the primitive propositions except that they be adequate for the deduction of the theorems. They are not independent propositions, but different forms of the same tautology.

Postulates, on the other hand, are subject to further conditions. They make significant assertions and define a system. It is possible to deny any one postulate in a well organized system, and by uniting the denial with the others, achieve a system which is different from the previous one. Mathematicians have usually considered three determinations of these postulates—independence, consistency and completeness, and have been content to accept systems whose postulates conformed to these conditions.

Should independence, consistency and completeness not themselves be independent, consistent and complete, one or the other of them would be derivable from or would be equivalent with the others, and therefore is not to be stated as a special condition; they would represent impossible, because incompatible, conditions which could not be fulfilled in any one system; or there would be further conditions which would have to be imposed on the postulates before they could be taken as containing within them the significance which is to be found in any and every theorem in the system which they determine.

That a set of postulates is made up of independent assertions may mean one of two things: 1. there are no duplicates of a given postulate; 2. there are no postulates which are derivable from one or more other postulates. If there are dependent postulates, that does not in any way effect the system. It means simply that we have unnecessarily multiplied our assertions, for we have provided $n + m$ premisses where there were only n assertions.

The duplicates can be discovered at sight and offer no difficulties. The elimination of derivable propositions is a much more difficult task. To discover whether one proposition is dependent on another, it is necessary to deny each postulate in turn, add the negative to the previous set and see whether the system remains consistent. Proof of independence thus rests on proof of consistency. *"Die Frage der Unabhängigkeit hängt mit der Widerspruchlosigkeit aufs engste zusammen; denn dass der Satz a von gewissen vorliegenden Axiomen unabhängig ist, kommt darauf hinaus dass der Satz—a mit ihnen nicht in Widerspruch steht."*[34] If the denial of A yields a proposition which implies the falsity of B, then by transposition B implies A, and A and B are not independent. Absence of independence does not invalidate the system, but simply denies that each postulate says something new. It is a stipulation of simplicity. It assures us that our set is truly basic, and enables us to distinguish a set of postulates from a set of theorems derivable from them and each other.

Huntington,[35] by making some improvements on Venn's diagrams has made it possible to diagramatically indicate whether a set is made up of independent postulates or not. He contends that any three postulates will divide the universe into 2^3, i. e. 8 compartments. If the postulates are independent, all are possible; if one is implied by some others, some will be empty. If we take three postulates, a, b and c, and if —abc is inconsistent, then as —ab implies —c, —ac implies —b, bc implies a, the compartment —abc will be empty. These diagrams are extensively discussed, though not in this connection, in Lewis'

[34] Weyl, *Philosophie der Math. u. Natur, Handbuch der Philosophie,* Abt. II, p. 192.

[35] "A new Set of Postulates for Betweenness," *Trans. Amer. Math. Soc.,* 28, 1924.

Survey of Symbolic Logic, Chapter 3. Another way of representing independence is mentioned by Weyl, in the paper just mentioned—the construction of a model, in which one endeavors to show that only some of the postulates are exemplified. Postulates are independent if in the exemplification of each the others are not to be found. The construction of models, however, brings in many additional considerations, not to mention the fact that it is not always possible to create them.

Consistency has three forms; 1. internal, 2. external, 3. deductive. The first has to do with the character of the individual postulate, apart from the others. It makes the postulate assertible or capable of having an ultimate meaning. The second makes possible the unification of diverse postulates in one system. It says that they all can be true at once. The third is irrelevant to the postulates, but has to do with the connection of the theorems with them; it is discovered by seeing whether the postulates together with the theorems which are derived from them state tautologies. The demonstration of the consistency of the other two is never really satisfactorily shown. We must either show that there is at least one instance which embraces all the postulates and is itself not self-contradictory, which amounts to the internal consistency of an instance, or prove that it is impossible to deduce from any one or more postulates the two theorems a and —a. What is implied by inconsistent premisses may be consistent, but what is inconsistent can never follow from what is consistent. A demonstration that all possible consequences are consistent, therefore, is necessary in order to demonstrate that the set is made up of consistent postulates. But when is it possible to know that the deduction has been carried on so far that

the possibility of inconsistent consequences has been eliminated? I have seen no such demonstration and know none. Is it possible that the only way we can determine whether a set is consistent is by seeing all the postulates actually exemplified in some one object? If so, we must arbitrarily assume that the object is self-consistent, so that the proof of consistency must ultimately rest on a dogma. As independence rests on consistency there are therefore no satisfactory proofs as yet of either independence and consistency.

Completeness has three forms: Perfect, absolute and relative. Perfect completeness is the ideal of knowledge, and of the universal system. It means that the set embraces every possible proposition. We have already discussed why this cannot be attained. Absolute completeness constitutes a set in which every possible proposition in terms of the given base is possible. One of every pair of contradictory propositions in terms of the base is implied by or expressed by the postulates. It can be verified only in a finite system where every alternate can be written down and checked off. Relative completeness defines a set. It is a condition of contentment. No deductions are granted to exist in the system unless they are contained in the postulates, and all relevancy or meaning is denied to any theorem not deducible from these postulates. It really is no condition at all, for it merely says that the postulates are adequate to the theorems which follow from them, and this must be true of any set of postulates. It achieves prominence only because we frequently know what we wish to demonstrate and know that a given set of postulates are inadequate for such a demonstration.

Independence is determined by denying each postulate in turn and showing that the other postulates are not

thereby denied; consistency requires an instance exemplifying all the postulates, and completeness follows tautologically from the definition of a set.

That independence is independent of consistency and completeness is seen from the fact that a system may be completely determined by one postulate, which is self-consistent, but may add other "postulates" which are derivable from it. And that which is derivable from a self-consistent postulate, must be self-consistent.

That consistency is independent of completeness and independence is seen from the set of two mutually contradictory postulates which are necessarily independent, and which completely determine a set of incompatible propositions involving the given elements and correlators.

Completeness is independent of consistency and independence if we do not permit our postulates to define the set. Thorems may be acknowledged, which though they do not contradict the postulates are nevertheless independent of them.

That independence, consistency and completeness are externally consistent can never be shown if proof of consistency requires an instance for it means that we shall have to prove the consistency of independence, consistency and completeness by means of a self-consistent system which itself requires a model. That this procedure is ultimately vicious should be obvious.

That they are relatively complete would follow from the fact that they are defined to be the minimal conditions of postulates. Such a completeness would be trivial. That they are absolutely complete is much more desirable and difficult to prove. If the laws of thought are complete, they should be complete, for they then express all the logical conditions that can be imposed on any entities.

But why should one believe that there is not a fourth "law of thought?" The best that can be done is to take them as absolutely complete necessary conditions; independence determines whether a given proposition is a postulate; consistency determines whether a set of propositions can determine a system, and whether a set of theorems can be in a given system, while completeness defines the system, and makes possible the exclusion of theorems which may conform to the principle of consistency. These three conditions then, defininig as they do the postulates, the system's theorems and the system itself, may be said to thus define the nature and meaning of the postulates in a system.

However, there are other requirements frequently imposed on postulates. Some times they are asked to guarantee the existence of at least one theorem, to relate to existents, etc. It is also possible to make refinements on the descriptions of the conditions of independence, consistency and completeness. Sheffer for example, distinguishes between minimal and maximum independence; the former involving the implication by A of a large part, but not all of B; the latter requiring that no postulate imply any part of any other. Consistency, Lewis has defined in terms of impossibility and negation. A. Church in an interesting recent article[36] has defined a set of postulates in terms of irredundancy. He holds that the necessary and sufficient condition for the irredundancy of a set of postulates is that they be independent and that the negatives of every two be contradictory. Irredundancy however (like the other refinements) does not offer a new condition, but is simply a special case of a combination of consistency and independence.

[36] "On Irredundant Sets of Postulates," *Trans Amer. Math Soc.*, 1928, Vol. 27, p. 318.

The postulates of any system are accordingly independent or dependent, consistent or inconsistent, complete or incomplete. As any one proposition is independent, and as the system is complete as far as it goes, we are left ultimately with the dichotomy of consistent and inconsistent postulates. Independence and completeness were necessary only to assure us that we had distinct postulates embracing the entire field. Independence is not necessary but desirable—it is an aesthetic criterion; completeness is necessary but unavoidable—it is a tautological criterion. The irreducible necessary condition for all systems, by which they can be significantly tested, is therefore, that the postulates be compatible; everything else is, strictly speaking, irrelevant.

XVI

Possibly the most important point to note in connection with systems, is that they are possible only because logic has already been assumed. This is an important point in connection with systems of logic. The laws of logic make possible the assertion of the laws of logic and the derivation of some laws from others. The specific laws of identity, contradiction, excluded middle determine every part of a system and make it possible for substitution and inference to take place. The conditions involved in the development of systems transcend the system and the laws of logic must be assumed before they can be deduced. When the *Principia* "deduced" *"the* law of the excluded middle," it do so only because it had already recognized its validity. All that was proved was that from the statement of other tautologies, this tautology could be derived, providing we may proper substitutions, and dissolved the entailment between tautologies wherever they

occurred. The rules that made possible this derivation were themselves tautological, so that logic was assumed in order to create a system of logic.

The whole significance of the entire set of theorems and primitive propositions in a logic can be exhausted by stating the nature of a tautology, and indicating the various ways in which it can be written. It is not necessary to systematize logic, and to do so requires the acknowledgment of everything beforehand. We should systematize when we wish to prove, but we can't prove the propositions of logic without first accepting them. The laws of logic are not contained within the assertions of them, and are not subject to themselves or to any other conditions. As specific statements, or as marks in an expensive book, they are subject to these laws. But the laws themselves are forever outside of such finite loci. To know logical laws in their immediacy, one must turn mystic and climb over their symbolic statements. One can only speechlessly point at the transcendent, knowing that "whereof one cannot speak, thereof one must be silent" . . . .

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DISCUSSION

SENSATIONS AND THE CONSTANCY HYPOTHESIS

WITH much in Miss Helen Smith's vigorous paper in the January *Monist* I heartily agree. Her discrimination between (a) sensible appearances, (b) sense-data, and (c) sensations is parallel to my discrimination (in *Mind and its Place in Nature*) between (a) perceptual data, (b) data of inspection or introspection, (c) sensations, mental states. Her criticism of Mr. Broad's identification of sense-data with "objective constituents" of perceptual situations also parallels mine.[2] But I think she is over-hasty in her rejection of the existence of sensations. (And I notice, with a malicious satisfaction, her consequent and confessed inability to suggest a solution for the epistemological problem!) Her argument against their existence seems to me clearly based upon a confusion. And it is just this one point to which I wish to call attention.

Sensations she defines as "mental events, or states, which follow upon the stimulation of a sense-organ, being generated by, and correlated with, the resulting nervous excitation. These entities are hypothetical."[3] This I accept, with the proviso that I believe sensations to be, actually, the very inner being of certain cerebral events, instead of being merely "correlated with" them.

She states what she calls "the constancy hypothesis," as follows: "Stimulation of an end-organ gives rise—it was conceived—to a new *element* in consciousness, and given a normal condition of the organism, the relation between the stimulus and the psychic

[1] Sensible Appearances, Sense-Data and Sensations, Vol. XXXIX pp. 99-120.

[2] Mr. Broad's book appeared while I was correcting the proofs of my book. I inserted several footnotes referring to his views. My footnotes on pp. 7 and 12 make the criticism which Miss Smith develops at much greater length in Part IV of her paper.

[3] Pp. 102 f.

entity is *constant*."[4] The view which she sets out to defend is "that the constancy hypothesis is false, and that therefore (a) there are no sensations—"[5]

To this I might reply that her "therefore" is pretty weak. Even if the "constancy hypothesis" is false, its falsity does not warrant her in saying that "therefore" sensations do not exist. But I want to say more than that. I want to point out the ambiguity which vitiates her discussion of this hypothesis. As formulated in the sentence quoted, it asserts a constant relation between outer stimulus and "psychic entity," i.e., sensation. But the arguments which she adduces against it are arguments against the existence of a constant relation between stimulus and *sensible appearance* or *sense-datum* (I use her own terms). Her argument collapses as soon as it is seen that the evidence which she offers is for the falsity of this second doctrine, whereas it would be only the falsity of the first doctrine which would have any tendency to discredit the existence of sensations.

Curiously enough, she criticizes Mr. Randle for failing to distinguish between sensations and sense-data, at the same time that she leans heavily upon his argument "that there is no introspective evidence for sensations;"[6] she finds his "main contention unaffected —namely, that there is no psychical element which is a function of a retinal impression, and that we cannot *find* any such entity either by introspection or inspection."[7]

Now it is true that "sensations" are not *discoverable*; she stated at the outset that they are "hypothetical entities." They are not facts of experience; they are assumed in order to account for the facts of experience. So the fact that we do not "find" them does not discredit their existence. What we *find* is—data of perception, inspection, or introspection, as the case may be. What her evidence, and Mr. Randle's evidence, tends to prove is that the constancy hypothesis is not true of these latter entities. "What we observe," she says, "is not a function of a physiological process." "There is now a great body of psychological evidence which condemns a

[4] P. 100.

[5] P. 103. The rest of the sentence reads—"and (b) the relation between sensible appearances and sense-data is what it appears." I am not quite clear as to the implications here, but I take this to be directed against Mr. Broad. At any rate, I am not quarrelling with this part of her contention.

[6] P. 103.

[7] P. 104. Her italics.

belief in a constant relation between stimulus and perceived entity—whether sensation or sense-datum. . . ."[8] But "what we observe" is *not* sensation; it is sensible appearance or sense-datum. And what this psychological evidence condemns is *not* the possibility of a constant relation between stimulus and sensation, but such a relation between stimulus and sense-datum, or between stimulus and sensible appearance.

The truth is, I take it, that "what we observe" varies not merely with changes in the outer stimulus but with changes in the reaction of the organism to the stimulus. I spent much space in my book defending this view. But this important aspect of my theory Miss Smith ignores. She seems to suppose that if there are sensations, produced in us by outer stimuli, "what we observe" must be thus completely determined. But to deny that was an important point in my theory. The physiological processes involved in sensing and perceiving include much more than the processes in the sensory nerves and the brain-centers directly reached by them; they include the extremely complicated reactive and associative mechanisms. The nature of the sensation directly caused by the stimulus is only one of the determinants of the sense-datum or sensible appearance. Her argument is thus persistently beside the point. She has, herself, made the distinction between sensation and sense-datum; but she does not seem to be able to keep the two kinds of entity distinguished, or to see that what is true of one may not be true of the other.

If, then, her argument against the existence of sensations collapses, are there positive arguments for their existence? Yes, I gave a lengthy exposition of my arguments in my book. Miss Smith refers briefly to these arguments, asserts that some of the phenomena which I adduce "can be explained equally well on some theories other than[9] mine, admits that two of my arguments would seem to her strong if she could accept the doctrine of "projection." But she finds that doctrine unintelligible; therefore she rejects *all* of my arguments.

Well, the doctrine of "projection" needs for its comprehension an understanding of this very motor psychology for which Miss Smith seems to have so large a blind spot. Neither the doctrine of "projection" nor the motor psychology is really very abstruse.

[8] Pp. 107 f.

[9] P. 110.

But a comprehension of the latter is essential for a comprehension of the concept of "projection." When one comes to realize the degree to which "what we observe" is dependent upon our *reactions* to sensory stimuli, one can understand how it is that in and by that reactive process, we "refer" or "impute" characters to the outer object. One then becomes more hesitant to assert that whatever one seems to see is really *there;* perhaps what really exists is—a set of sensations in one's head and one's body reacting in complicated ways to the surrounding world. Perhaps these floating shapes and sounds which we call sensible appearances and sense-data are but a confused blending of our apprehension of inner sensations with our reaction to outer stimuli.

To reach such a point of view requires long and patient thought. I am not going to inflict upon the reader even a summary of the elaborate argument given in my book. I wish to point out merely that Miss Smith's argument *against* my theory rests upon a failure to keep clearly in mind the distinction which she herself makes, between sensation and sense-datum or sensible appearance.

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SOME RECENT BOOKS

An Anthology of Recent Philosophy: Selections for Beginners from the Writings of the Greatest 20th Century Philosophers. Compiled by Dr. Daniel Sommer Robinson. New York, Thomas Y. Crowell Co., 1929. Pp. vi+674.

This first anthology of contemporary philosophy gives an excellent cross-section of the thought of our time. Dr. Robinson's selections are representative and carefully chosen. There is a major grouping into five parts. Part One deals with the general nature of philosophy and its present day divisions. Part Two contains selections from the idealistic philosophers. Personalism and Absolutism are distinguished. Part Three deals with realism. The articles represent the positions of neo-realism and critical realism. Part Four is the section on pragmatism and Five contains excerpts from the writings of Driesch, Vaihinger and Spengler, together with discussions of emergent evolution, mechanism, vitalism, neo-scholasticism and the philosophies of Spencer and Nietzsche. The appendix gives brief, but valuable biographical and bibliographical accounts of contemporary philosophers. The sub-sections on realism, idealism and pragmatism group the discussions into solutions of the problems of Knowledge and Existence, Truth and Error, Body-Mind, and Value.

Dr. Robinson's analyses at the beginning of each article, while probably valuable to the lay reader, are not advantageous from the student standpoint, as he has done admirably and exactly what the student should work out. The questions at the end of each chapter are frequently significant only when considered in connection with the author's view as a whole, and cannot be answered adequately on the basis of the article alone.

The book in no sense is "for beginners" as its sub-title would indicate, but should prove helpful as a general reference book, and can be used in advanced classes, where the philosophical positions are already understood in general.

Realism: An Attempt to Trace its Origin and Development in its Chief Representatives. By Syed Zafarul Hasan. D. Phil. (Oxon.) Cambridge University Press, 1928. Pp. xi+333.

The introduction to this book gives in outline Dr. Hasan's own philosophical view as it has developed partly from his critique of realism. Starting with a definition of realism as "the doctrine which maintains that the external world exists and is directly apprehended in perception," he develops the origin and beginnings of the theory. There are special sections on Descartes, Locke, Reid and Hamilton, Schuppe, Mach and Avenarius, Meinong, Stout, the Critical Realists, Cook Wilson, Prichard and Joseph, Alexander, Holt and Russell. But the *Alpha* and *Omega* of modern realism is to be found in the writings of G. E. Moore. Dr. Hasan devotes fifty pages to the historical development of Moore's thought.

The author's presentation of the different realistic positions is clear and his criticism keen. The book is difficult reading, not only on account of the subject matter, but due to its lack of style. It would probably be of interest to only the professional philosophers and particularly to those interested in epistemology.

New Realism and Old Reality: *A Critical Introduction to the Philosophy of the New Realists.* By D. Luther Evans, Ph.D. Princeton, The Princeton University Press, 1928. Pp. 214.

As any philosophical movement grows it is necessary from time to time to draw together its various doctrines. Dr. Evans skilfully accomplishes this in this volume. In Part One he gives the background and development of realism. Part Two and Three combines the neo-realistic views in a system under the various sub-divisions of ontology, cosmology, epistemology, theology, axiology and psychology. The appendix gives an excellent bibliography of the American and foreign realists together with a list of the critical discussions of recent realistic philosophy. There is an adequate index. The book should prove interesting to the general reader in philosophy and supplemented by readings could be used for class room discussion.

Personality and Immortality in Post-Kantian Thought. By E. G. Braham. London: George Allen and Unwin, 1926. Pp. 246, Price 7 s. 6d. net.

In one sense this volume is intended as a sequel to an earlier work, *Problem of the Self and Immortality*, which traced the same subject from Descartes to Kant. The author feels that many books on Immortality pass by a basic initial problem, namely, the nature of the self. And while psychology may be a very fruitful angle of approach, the core of the problem is metaphysical. The conclusions of this study, which may be said to form the third and last division, are briefly as follows: First, human personality is a unified system of experiences which begins at a very low level but reaches a high degree of complexity and value; secondly, the rational, moral and religious nature of the self unites in a "demand" for its infinite duration in time; thirdly, the unity and stability of the universe, with freedom, is more adequately met byTheism than in either a monism of the general Hegelian type or a pluralism as in James. "This theistic view saves human personality from absorption into an Absolute and also delivers it from the clashing elements of an unsatisfactory pluralism wherein is no belief in a dominant unifying principle or person in the universe." (page 227)

Of the monistic type the author gives a concise analysis of McTaggert, Bradley, and Bosanquet. Pluralism is represented by James and Ward. The critical portions of the volume are based upon extensive studies and the results are stated in extremely compact form. The net result may be stated briefly: The self is a real unified system of experiences that develop *in* time but is not *of* time. It continues to develop in a trans-mundane world. Such are the pointings of moral and religious experience. But in the end, this conclusion rests back upon trust in a God of love.

Plato: *The Man And His Work.* By A. E. Taylor. New York, Lincoln MacVeagh, 1927. Pp. ix+520.

This work meets a real need. Coming as this does from the pen of one of the greatest living authorities on Plato's life and work, and a lifelong student and expositor, it will be hailed with delight by all lovers of philosophy. No work of equal scope and completeness has hitherto been available in the English language. Each of the dialogues is analysed in detail in the light of its historical background and its relation to the other dialogues. Taylor has admirably succeeded in accomplishing the aim he set himself, namely, "to tell the reader just what Plato says," and not force any "system" upon him. And this he has done not only with masterly scholarship and a sure knowledge of the questions involved, but also in such a clear and interesting style that the average reader with a taste for philosophy but no great technical erudition is certain to find the book a very readable one, and an excellent companion to the dialogues themselves.

La philosophie morale de Josiah Royce. By MOSES JUDAH ARONSON. Paris, Librairie Felix Alcan, 1927. xv+185. 25 francs.

This lucid monograph presents the moral philosophy of Royce as the fruit of a specific epoch and milieu. The central theme of Royce's ethics is said to be loyalty, conceived as the fulfillment of the potentialities of the individual through devotion to a universal community, a social and spiritual order inclusive of all rational beings. Thus the thought of Royce acquires a deeply religious cast. Now, even the political organization of America is an *e pluribus unum.* Beginning with an account of the solidarity of the individualistic Puritans in the church of Leyden, the author carries his theme throughout the life of the colonial theocracy, and exhibits the same motif in the Constitution of 1788 and in the thought of Channing, Lincoln, Emerson and Whitman. All of this, and what is described as the resulting philosophy of Royce, is set forth in terms too single and too simple, but the work as a whole, while of little value to technical philosophers, should prove interesting to the general reader.

Purposive Evolution. By Edmund Noble. New York, Henry Holt & Co., 1926. Pp. xi+578. $4.50.

This volume, which includes, in substance, a number of papers previously published in the *Monist* and in the *Open Court,* is an extraordinarily careful analysis of the problem as to whether evolution is a process whose results are attributable to the haphazard clash of forces and materials or whether it exhibits a principle of direction. "Are we to assert, as in the conventional view, that purposiveness can show itself only as an outcome of and as associated with consciousness, will and life, or is not purposiveness to be recognized in advance of these as the pre-condition, the very *sine qua non,* of evolution?" The book carries as its subtitle, "The Link Between Science and Religion." This link is found in cosmic purposiveness. Now hitherto science has failed to recognize this purposiveness because, as Mr. Noble contends, science has simply taken for granted the organism and its life powers; religion, on the other hand, has clung to psychomorphic, even to anthropomorphic, views of design in nature, thinking of the latter in terms of man's experience of the way in which he impresses his will upon objects. But the cosmic trend, the author urges, should be neither ignored nor humanized. Meanwhile, "sympathy with the scientific attitude should help potently to unify the religious outlook, still broken up by differences of race and denomination, just as sympathy with the religious point of view should aid science in realizing the limitations of knowledge and the impossibility of regarding it as a universal solvent."

Problems of the Pacific. Edited by J. B. Condliffe. Chicago, The University of Chicago Press, 1928. Pp. xiii+630. $3.00.

During July of 1927, one hundred and thirty-seven influential persons from the various countries that border upon or exercise power over the Pacific met in Honolulu as the Second Conference of the Institute of Pacific Relations. Though doubtless well informed regarding their governments' policies, the members of the Institute spoke for themselves alone and thus exhibited a frankness and a freedom scarcely possible to anyone in an official capacity. These consideratons were strengthened by the fact that the objective of the Institute was not the enactment of resolutions or the direct exercise of political influence but was mutual clarification and improved understanding. The published proceedings thus form a document of prime social and, indirectly, political importance. Opening statements give the outlook on Pacific affairs of each of the various national groups; there are summaries of round table discussions and thirty-three essays dealing with the most vital and thorny issues touching the national and racial interests in connection with the problems of the Pacific.

Speech: Its Function and Development. By Grace Andrus deLaguna. New Haven, Yale University Press, 1927. Pp. xii+363. $5.00.

Speech is here treated not as an expression of ideas but, to an important extent, as a phase of the organized social life, having the function of co-ordinating the activities of the members of the group. As regards origin, it is argued that "the change from arboreal life to ground-dwelling must have made a more flexible type of group organization highly advantageous, if not indeed necessary, and it was probably in serving this end that speech developed." Utilizing a behavioristic psychology in discussing the function of speech in the life of the individual, the author sees in speech "a further extension of the indirectness in the attainment of individual ends which has marked the development of intelligent behavior from the beginning. The higher mental activities—conception and purpose, memory and imagination, belief and thought—so far as these are distinctively human, are found to be closely dependent on speech. They are fundamentally social in origin, being due indirectly to the development of *conversation*, which, it is argued, has the primitive function of preparing for concerted group action, much as distance-perception prepares the immediate response of the individual. Conversation is shown to have a characteristic structure, adapted to its function, and it is this structure which makes possible the organized activity of thought, in which it is reflected."

The Scientific Habit of Thought: An Informal Discussion of the Source and Character of Dependable Knowledge. By Frederick Barry. New York, Columbia University Press, 1927. Pp. xiii+358. $3.50.

This book admirably reflects the mind of one holding the office which the author occupies—he is Assistant Professor of the History of Science in Columbia University. Very lucidly there is set forth, essentially from the point of view of instrumentalism, the logic of science and the philosophy underlying scientific method. This discussion is followed by an account of "The Nature of Fact," and then by a third section on "The Elements of Theory," relating primarily to hypothesis and law, the fundamental importance of mathematics in scientific theory, and scientific synthesis. Section VI, entitled "Scientific Humanism," sets forth the role of science in modern culture and advances suggestions with reference to the proper place of science in general educational curricula.

The Science of Religion: An Introduction. By Lewis Guy Rohrbaugh. New York, Henry Holt & Co., 1927. Pp. xii+291. $3.00.

This book, though not without elements of value, is in fact not true to its title. Not only does it fail to offer any contribution to our scientific knowledge but it introduces, without regard to scientific method or to an adequate discrimination between the value of its sources, much material derived from sheer speculation or from the devotional and historical literature of religion. The author exhibits an obvious concern to defend, through arguments of various sorts and of differing degrees of scientific and logical cogency, conceptions and positions of an essentially traditional character.

Current Christian Thinking. By Gerald Birney Smith. Chicago, The University of Chicago Press, 1928. ix + 209.

This book contains a clear and fair account of the present day Christian thinking. It lays a foundation with an account of the Roman Catholic and early Protestant thinking and is followed by a discussion of modernism as it first appeared in the Catholic Church and how it was dealt with there in 1907 by the well known encyclical letter. There follows an account of modernism in the various Protestant churches and how it called up opposition in the form of fundamentalism. The rest of the book is devoted to an almost enthusiastic but well stated acount of evangelical Christianity.